An abrupt stochastic damage function to analyse climate policy benefits*

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Abstract

This paper studies uncertainty about the non-linearity of climate change impact. The DIAM 2.3 model is used to compute the sensitivity of optimal CO2 emissions paths with respect to damage function parameters. This builds upon results of the EMF-14 uncertainty subgroup study by explicitly allowing for the possibility of threshold effects and hockey stick damage functions. It also extends to the cost-benefits framework previous studies about inertia of energy systems. Results show that the existence of a threshold in the damage function is critical to precautionary action. Optimal path are much less sensitive to uncertainty on the scale of the damages than on the threshold values.

1 Introduction

This paper examines optimal CO_2 abatement policy using a coupled model of climate and economic dynamics under uncertainty. First, we argue that the importance of precaution is magnified by the fact that abrupt changes are likely to happen. We show numerically that this kind of uncertainty consideration cannot be neglected to specify correctly the cost-benefits analysis of climate-energy policies. Second, we show more precisely that uncertainty about the magnitude of climate impact is far less critical than uncertainty about the date at which they could occur.

Our analysis builds upon four simplified beliefs about the danger of climate change: no attributed damage today, small expected future impact, small risk with large consequences and expected arrival of information. In more details:

1. The magnitude of negative socio-economic consequences presently attributed to climate change today is small in front of the measurement errors and the interannual variability of social welfare indicators due, for example, to business cycles and weather variability.

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2. In expected value, most analysts foresee only a modest direct impact of climate change on economic growth over the course of this century, because most of the value-added in the global economy occur in sectors relatively insensitive to climate change, and adaptation is possible in other sectors.

3. Greenhouse gas forcing in the 21st century could set in motion large-scale, high-impact, non-linear, and potentially abrupt changes in the physical and biological systems over the coming decades to millenia, with a wide range of associated likelihoods. Many natural and managed ecosystems may change abruptly or non-linearly during this century.

4. Significant progress is being made on understanding climate change and human responses to it. However, there remains important areas where operational knowledge is decades away, such as the quantification of impacts at the local level, the effects of adaptation and mitigation activities, the definition of sustainable development or what constitutes "dangerous anthropogenic interference with the climate change".

While these beliefs do not constitute *robust findings* in the meaning of [IPCC, 2001, p.30], they nevertheless represent the present outcome of over a decade of research about the impacts of climate change. Together, they lead to several difficulties in trying to justify on economic terms policy actions like the Kyoto protocol. Historically, attempts to do so can be reviewed as a movement from deterministic cost-efficiency analysis to stochastic cost-benefit analysis.

Early assessments highlighted that if the atmospheric carbon dioxide (CO_2) concentration is to be stabilised at a level of 450 ppmv or below, economically optimal strategies imply significant abatement of carbon emissions in the short run. On the other hand, if the ultimate concentration target is over 550 ppmv, then models show that the cost of deferring abatement by a decade or two is not very large. Thus, given these results, the near term mitigation objectives are tied with ultimate CO_2 concentration target. Here the difficulty is to justify which target to aim at, given that reasoning only with expected damages is obviously irrelevant to precaution against the risk of abrupt climate change.

Then in Ha-Duong et al. [1997] we assumed an unknown concentration ceiling of $\{450, 550, 650\}$ ppmv with equiprobability, and found that significant near-term abatement were economically optimal. On the other hand, with thresholds from 550 to 850 ppmv, Yohe and Wallace [1996] found modest optimal abatement response over the next several decades. This illustrates the difficulty in this kind of a stochastic framework: the results depends upon the considered concentration targets and their probability, especially on the lowest target. As [IPCC, 2001, p. 350, figure TS-10a] notes, the degree of near-term hedging in this analysis is sensitive to the fact that the ultimate target must be met at all cost.

This is why we have to turn next to cost-benefit analysis, and examine models without pre-defined CO_2 concentration stabilisation target. In a cost-benefit optimum, pollution is reduced to the point where further additional abatements do not bring benefits larger than their costs. Results of cost-benefit models shown in [IPCC, 2001, p.

350, figure TS-10b] suggest that the optimal hedging strategy against a low-probability, high consequences climate risk is very close to no hedging at all.

In our view, this gap between cost-efficiency models results —early abatement can be valuable— and cost-benefit models results —optimal early trajectory close to the reference case— can be explained by damage function specification problems in the later. Using an S-shaped damage function can change the result of the cost-benefit analysis.

Section 2 discusses in more details results of the existing literature, which has recognised the importance of surprises and non linearities in the climate change issue at the theoretical level. In our opinion previous numerical models have found surprising results because they misspecified the four stylized beliefs enumerated above. Section 3 describes a nonlinear, stochastic climate damage function used to run a cost-benefit version of the Dynamics of Inertia and Adaptability Model (DIAM 2.3). Section 4 discusses the sensitivity of optimal short-term policies to the shape of the damage function. It demonstrates that an abrupt damage function implies a larger near-term abatement policy, and that this result is more sensitive to the date of the nonlinear change than to the magnitude of the catastrophe.

2 Methodological issues

2.1 Climate change impacts and uncertainty

Analysing the geophysical consequences of climate change remains a very speculative science. Analysing the economic and human consequences is even more so. Tackling both together to assess the impacts of climate change is one of the biggest difficulties of climate policy analysis. It is therefore not surprising that the representation of the risk is one of the least convincing components of long-term energy-climate policy models.

The relationship between climate change and its impact on human welfare is conveniently discussed with the concept of an impact function. This function is mathematically formalised as $D = f(M)$, where M denote the magnitude of climate change and D represent its social welfare impact. The level of change M could be defined as global warming ΔT in degrees. It could otherwise be the increase of the radiative forcing in Watts per square meter, or also the increase of atmospheric carbon dioxide concentration. The damage D includes market and non market impacts. It is usually represented in monetary units, divided by the Gross World Product to have a dimensionless number, so it can be read as a fraction of GWP lost at a given date.

For example if $D = 2\%$ twenty year from now, this can be compared to a decrease in the world growth rate from 1.5% to 1.4% during twenty years. This commonly used order of magnitude, one tenth of a percentage point of growth during two decades, fits the stylized belief that damages are small compared to interannual variability.

In its simplest interpretation, f is an increasing function between a real-valued change $M(t)$ and the real-valued impact $D(t)$. This overlooks many essential characteristics of the climate change issue: the issue of the rate of climatic change, that of inter-annual variability, and that of the risk of abrupt large-scale change in the climate system.

- The rate of climatic change is important when it comes to the question of the adaptability of ecosystems and societies. It may turn out that controlling that rate dM/dt is more important, with respect to the near-term policy, than the ultimate long-term greenhouse gases stabilisation level.

- Changes in the inter-annual variability of climate are important. While climate and climate change are defined as averages over a time period several decades long, physical variables underlying $M(t)$ (such as temperature or precipitation) are a rapidly varying stochastic function of time. Dalton [1997] has shown that introducing the second moment of $M(t)$ in the damage function leads to greater climate change effects than without.

- The earth system is known to be nonlinear, therefore abrupt changes in climatic conditions can happen. This is potentially leading to a perceived rapid increase in economic losses. The classical example of such a non-marginal change in the climate system is the collapse of the north-Atlantic thermohaline circulation described in Broecker [1997].

This paper considers M as an aggregate environmental indicator of climate change that implicitly represents changes in the rate and in the variance, in order to focus on the third point and represent explicitly the risk of abrupt changes.

Gjerde et al. [1999] remarked that in the literature, modeling of optimal climate policies given the possibility of a catastrophe has been done using either one of the two following approaches: continuous-time real option models solved analytically, or stochastic optimal control models solved numerically.

In continuous time, there seems to be no consensus about the sign of the quasi-option value to reduce emissions. Dixit and Pindyck [1994]'s real-option model found that more uncertainty implies to delay emissions reduction further. The interpretation of this result is that investment to reduce emissions is more irreversible than greenhouses gases accumulation. But these results assume a linear damage function. Narain and Fisher [1998] showed in a model with an avoidable climatic catastrophe explicitly included, that the environmental irreversibility effect could be stronger than the investment effect.

The research presented in this paper also explicitly models an avoidable climatic catastrophe, but uses numerical discrete time stochastic optimisation. It is in essence an expansion of the previously published DIAM model. It was initially motivated by the necessity of introducing cost-benefit in the analysis as discussed in introduction, and also by a couple of surprising findings arising from previous modeling exercises, namely DICE and the EMF-14 uncertainty studies.

2.2 Damage function specification: two issues

DICE's damage function is $D \approx \theta_1 (\Delta T)^{\theta_2}$. The base value $\theta_2 = 2$, as discussed by Nordhaus [1994], has greatly influenced subsequent studies, although factually very little is known about it. Table 1 exhibits the sensitivity of optimal emissions reduction to a doubling of either θ_1 or θ_2 in the damage function.

DICE model parametrisation	Optimal abatement % of global CO_2 emissions	
	in 1995	in 2095
Base case	9.0	14.3
Doubling damage function intercept θ_1	13.0	20.5
Doubling damage function exponent θ_2	8.9	25.9

Table 1: DICE sensitivity of optimal carbon abatement levels to the impact function parameters, from [Nordhaus, 1994, table 6.4 page 109].

Results depend significantly on these unknown parameters. Increasing the exponent θ_2 has a big positive effect on the long run optimal abatement, but a small negative effect on the short run. This negativity disappears when θ_2 is pushed further, for example with $D = .027(\Delta T/2.5)^{12}$, the optimal climate policy for the 1995 period is a 17% abatement. The table suggests that the near term optimal emission reductions appear more sensitive to the scale parameter θ_1 than to the exponent of the damage function.

To some extend, these results can be surprising, as they go in a different direction from Peck and Teisberg [1993] results on the importance of nonlinearity. Using the general case $D \approx \alpha(\Delta T)^{\lambda}$, with λ being 1, 2 or 3, their computations demonstrated that results were more sensitive to the exponent of the damage function λ than to their absolute magnitude α.

The second set of surprising results arise from a comparative study on uncertainty described by Manne [1996], led in the Energy Modeling Forum 14. The study was a comparison of seven climate/energy integrated assessment models with stochastic damage functions. One of the main focus of interest was to compare the results between two standardised runs.

- First, in the base case using the model's $D = f(\Delta T)$ damage function.
- Second, in a potentially catastrophic scenario, where there is a 5% probability that the damages are multiplied by 7.8, therefore having $D = 7.8f(\Delta T)$ as the damage function.

As shown in table 2, it appears that the hypothesis of a catastrophe had very little, if any, effect on the optimal near-term abatement level in these models.

These results are surprising with respect to the intuition that models should be sensitive to the possibility of non-marginal changes. However, two criticisms can be made to the representations of the impacts discussed above.

First, multiplying the scale of the damage function quickly leads to excessive damages. For example, if a quadratic function is calibrated so that 1 degree Celsius warming implies 1% of damages, then the corresponding impact at 3.5 degree warming is 12.25%. Under these assumptions, a factor of 7.8 on the damages leads to an almost total (>95%) economic disruption: the model is out of its limits. This is in contradiction with the belief that damage will remain relatively small.

	Optimal CO_2 emissions world GtC, year 2000	
	Without	With
Model	catastrophe	catastrophe
CETA	6.51	6.50
DICE	7.46	7.45
DIAM	6.99	6.99
HCRA	6.85	6.84
MERGE	6.66	6.66
SLICE	7.15	7.14
YOHE	7.25	7.14

Table 2: Inter-models comparison of optimal CO_2 emissions (GtC in year 2000), with and without a catastrophe in the model (damages multiplied by 7.8 with 5% probability, catastrophy occuring and observed from 2020 onwards). Surprisingly, the table shows that these models' near-term optimal results are not sensitive to the possibility of a climatic catastrophe.

Second, increasing the exponent ($\theta_2 = 1$, 2, 3, 4 or 12 have been quoted in the literature) increases the curvature of the damage function everywhere, including near zero. This leads to the paradoxical consequence that the larger the long-term damages, the smaller the short-term impacts. This effect explains the negative relationship in table 1 between θ_2 and near-term abatement: 8.9% when $\theta_2 = 4$ versus 9.0% when $\theta_2 = 2$ per cent. Moreover, the exponent increase leads even faster to excessive damages levels.

The hypothesis examined in this paper is that a more non-linear representation of climate change impacts that avoids these two criticisms also contributes to bridging the gap between results and intuition.

3 Model

The DIAM model 2.3 has four non-linear equations and three linear constraints. It is coded in the GAMS language and can be examined at the author's electronic homepage[1]. Because DIAM was previously discussed in Ha-Duong et al. [1997], Ha-Duong [1998], this section only briefly describes the model, and then focuses on the modifications made to the damage function.

The model finds an optimal strategy that maximises the discounted sum of intertemporal utility of the production W_t. The control variable is the reduction level X_t of carbon dioxide emissions at period t, defined so that the realised global emissions are $E_t = E_t^{ref}(1 - X_t)$. The social objective at each period is the logarithm of production. Production at each period can be affected by two factors: the cost of emission reductions, and the climate change impact. The reduction costs depends directly on

[1] At http://www.centre-cired.fr/. This model is available under the terms of an open-source licence: the published code can be re-used, modified and redistributed.

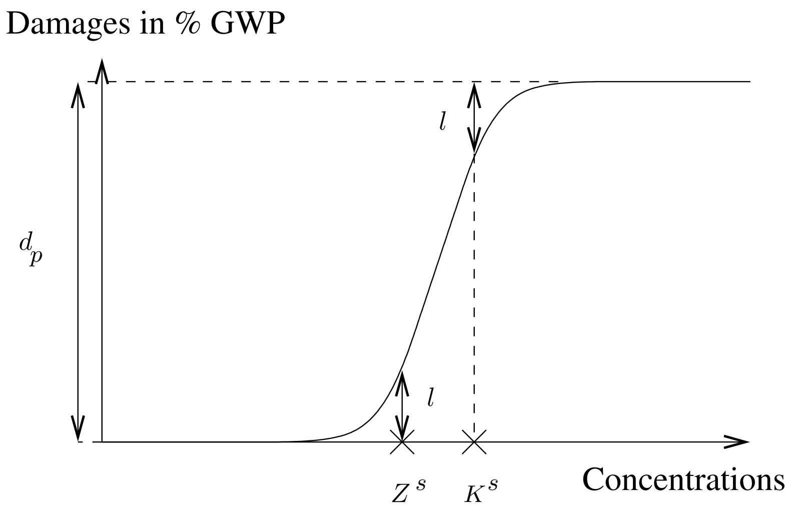

Figure 1: Non-linear term g_t^s of the impact function. The jump occurs over the interval $[Z^s, K^s]$. The ceiling d_p was set to 4% of GWP. Abruptness of the jump is parametrized by the $\gamma = l/d_p$ ratio.

the abatement level X_t and the abatement speed $X_t - X_{t-1}$. The climate change impact depends indirectly on X_t through a carbon-cycle model relating linearly carbon emissions and carbon dioxide atmospheric concentration.

Uncertainty about climate damages is represented by a subjective probability distribution over three possible states of the world s, denoted respectively L, C et H, and corresponding respectively to low, central and high climate change. Initially, the state of the world is uncertain, but it is known that information will arrive as follows: in 2040, that is period 6 in the model, one knows whether the state is H or not; in 2060, period 8, the information is complete in all cases.

Therefore, the model's output is an optimal global CO_2 strategy which depends upon the received information. Before 2040, only one trajectory is prescribed. Between 2040 and 2060 there are two branches: one corresponds to the optimal path in the H state, the other branch corresponds to the other two confounded states of the world. After 2060 at last, each one of the three branches corresponds to one then-known state of the world.

The nonlinear, stochastic damage function depends upon CO_2 concentration only. As a fraction of reference production W_t at date t, climatic impact is the sum of two terms f and g, that is:

$$B_t^s = (f_t^s + g_t^s) W_t \tag{1}$$

The first term f is, as in most models, a power function. Its magnitude is parametrised by θ_1^s, which represents the damage occurring at a doubling of CO_2-equivalent radiative forcing. The concavity depends upon the exponent θ_2^s. These parameters θ_1^s et θ_2^s

are lower in the L state of the world, and larger when the state of the world is H.

$$f_t^s = \theta_1^s \left(1-\sigma\right)^{t-t_0} \left(\frac{M_{t-L}^s - M_0}{M_{2x} - M_0}\right)^{\theta_2^s} \tag{2}$$

In equation (2), the lag $L = 30$ years represents oceanic thermal inertia. Impact is set to zero in the first period, which correspond to a lagged CO_2 concentration of $M_0 = 314$ ppmv. Over time, the impact function's scale declines exponentially at a constant rate $\sigma = 1\%\mathrm{yr}^{-1}$ to represent adaptation and structural change in the economy. The linear part of the damage function is scaled by reference to a doubling of pre-industrial CO_2 concentration, that is $M_{2x} = 550$ ppmv. At this level, damage is assumed to be $\theta_1^C = 1.5\%$ of the Gross World Product (GWP) in the central state of the world. The alternate two values for θ_1^s also corresponds to the IPCC estimates of a few percent of GWP. With respect to θ_2^s, values 1, 2 and 3 will be considered following the literature.

The second term g of damages is the threshold function represented figure 1. This term increases from practically zero to the level d_p over a concentration interval $[Z^s, K^s]$. The nonlinear jump was set to a significant $d_p = 4\%$ of Gross World Product. The economic interpretation of a 4% damage per decade can be understood as follows. In the context of a global economy expanding at 2% per year in the reference case, that damage occuring is equivalent to saying that to the global economy grows only at 1.8% per year during a span of 20 years.

$$g_t^s = \frac{d_p}{1 + \left(\frac{2-\gamma}{\gamma}\right)^{\left(\frac{K^s + Z^s - 2M_{t-L}^s}{K^s - Z^s}\right)}} \tag{3}$$

The thresholds parameters were set as figure 2 illustrates. The intuitive story is that the climatic system undergoes a transition process when carbon dioxide concentration rises from Z^s to K^s. The nature of this transition is not explicit in the model. The costs are presumed to represent effects of increased climate variability and the costs of adaptation to the new climatic conditions.

To guesstimate Z^s and K^s, we assumed that the non-linear transition occurred as the long-term equilibrium global warming passed through the [+3.5, +4.5] degrees Celsius range. In terms of the global warming observed at date t, this corresponds to levels much lower than 3 degrees C, since it takes decades to reach the thermal long-term equilibrium.

Since in the model the state variable is carbon dioxide concentration M_t^s, this temperature range was mapped back into a concentration range using a proportionality coefficient. This coefficient ΔT_{2x}^s is the temperature sensitivity parameter, and depends upon the state of the world. States L, C and H respectively correspond to values +2, +2.5 and +3.5 for ΔT_{2x}^s.

The influence of the shape of the damage function on optimal emissions was examined using the model DIAM. More specifically, three runs were compared.

linear In this run, uncertainty is set upon the slope of the damage function $\theta_1^s = \{0.5,\ 1.5,\ 4\}$ percent. The function f is linear, $\theta_2 = 1$. There are no catastrophe, so that $g = 0$.

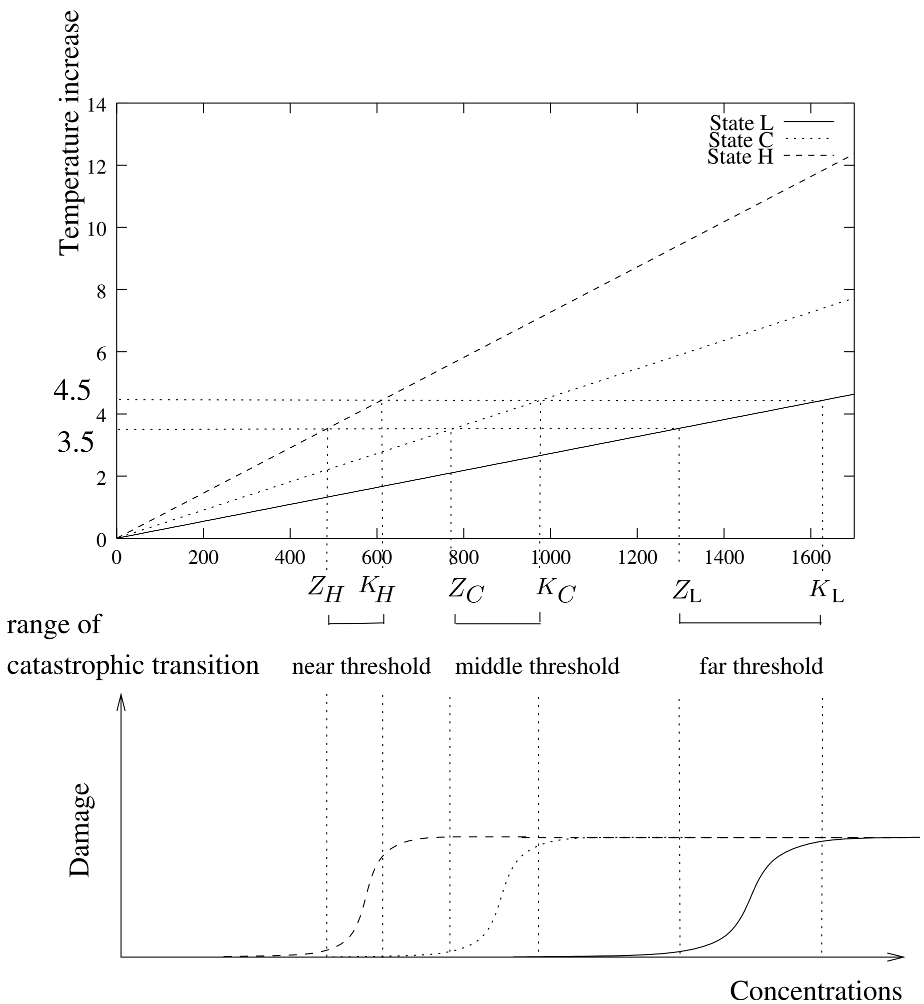

Figure 2: Empirical estimation of the non-linear climatic impact jump interval. The top panel's three sloped lines represents a linear relationship between global warming (vertically) and CO_2 increase (horizontally) for three different values of the temperature sensitivity parameter ΔT^s_{2x}. The horizontal lines at +3.5 and +4.5 degrees Celsius represents the interval over which the climatic system bifurcation occurs. For each sloped line, this (vertical) temperature interval defines a (horizontal) CO_2 concentration interval $[Z, K]$. The bottom panel displays g, the nonlinear jumps in the damage function.

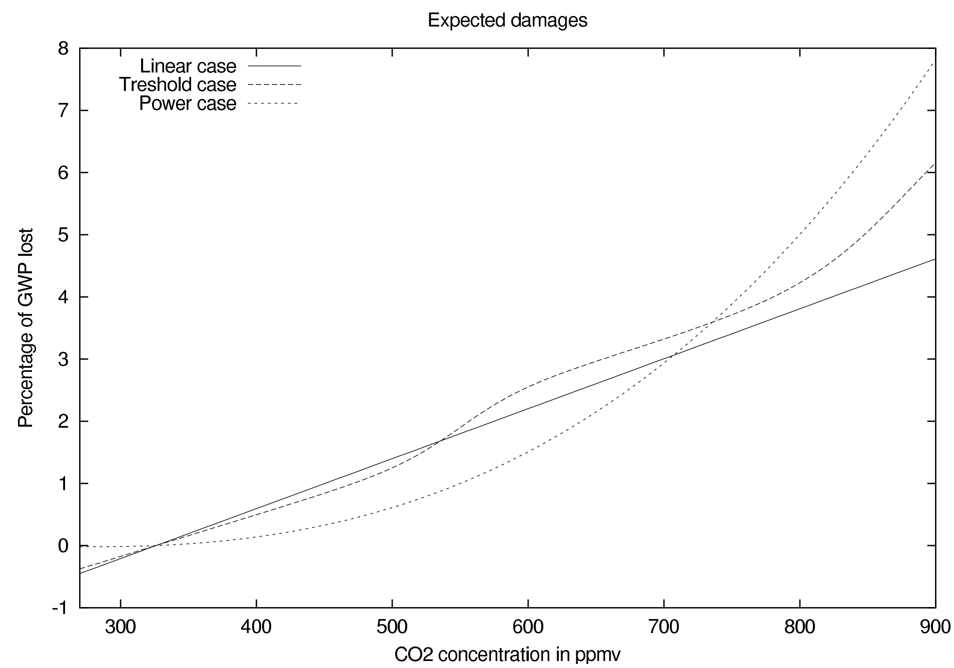

Figure 3: Expected climatic damage for three different shapes of stochastic functions. In the linear case, uncertainty is on the slope θ_1^s of the damage function. In the power case, uncertainty is on the exponent θ_2^s. In the threshold case, uncertainty is on the threshold Z^s.

power In this run, uncertainty is set upon the power of the damage function, so $\theta_2^s = \{1,\, 2,\, 3\}$ and $\theta_1 = 1$ per cent. There are still no catastrophe, $g = 0$.

threshold In this run, uncertainty is set upon the threshold at which the catastrophe occur, $\theta_1 = 1$ percent, $\theta_2 = 1$ and $Z^s = \{481,\, 770,\, 1283\}$.

Figure 3 displays the average of these three damage functions. As it was trivial that, all other things being equal, a larger expected damage leads to larger abatement, the different damage functions are calibrated to keep approximately constant the expected value of the damage.

In addition we sought to understand the critical drivers of the abatement strategy in the threshold case. To this end a sensitivity analysis on the threshold function parameters has been conducted. Sensitivity to the ceiling height and the abruptness of the step were examined.

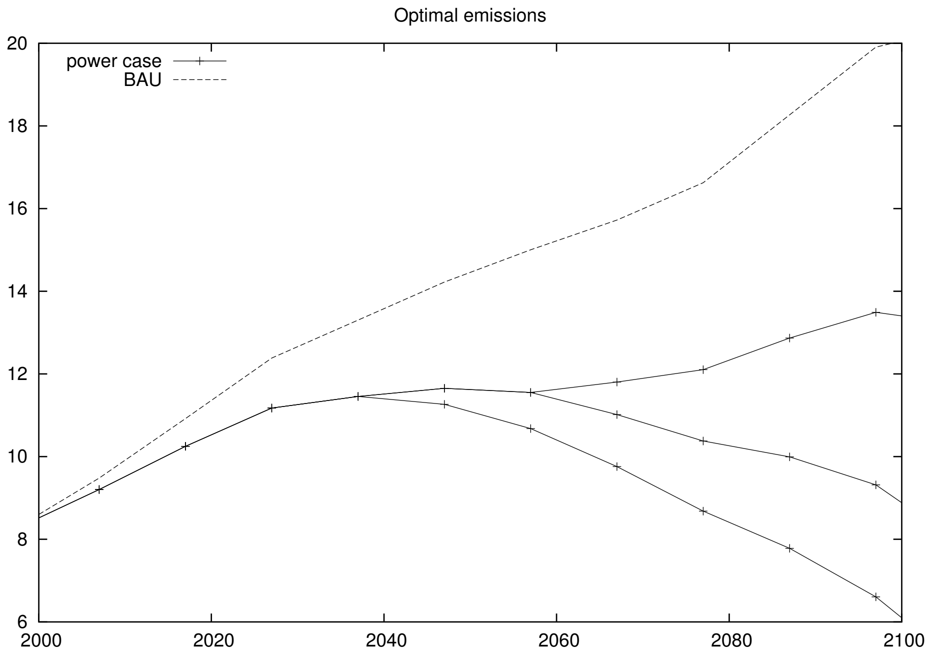

Figure 4: Optimal CO_2 emission path for the power stochastic impact function, and the reference business as usual case.

4 Results

The model computes optimal carbon emissions trajectories responding to different assumptions about the climate change risk. Three assumptions were compared, using three different shapes of stochastic damage function: threshold, linear, power. In each case, we examined two parameters: the short-term emission reductions and the effects of learning.

Figure 4 presents an optimal emission paths and the reference business as usual case. In the reference case, the emissions grow more or less linearly. In the other case, two bifurcations occur, corresponding to the learning dates. These results show a stabilisation of the world emissions in about 2050 when the state of the world is the Central state C, but they decrease as early as 2040 in the High state of the world.

Comparing the different optimal emission pathways associated with the different damage functions, the central result is that in the threshold case emissions are lower than in the other two cases. Detailed results for 2020 appear in the table 3. They show that the effect of uncertainty on θ_1 (scale of the damages) or on θ_2 (exponent of the damage function) are comparable in order of magnitude, while in the threshold case the percentage of emission reductions are 1.5 points higher than in the two other cases. This result is also valid in the mid term, as that difference grows to 2% in 2030 and 3% in 2040.

	Abatement (%)	Emission (Gt C)	Concentration (ppmv CO_2)	Abatement costs (%)
Baseline		10.91	414	
Linear	7.2	10.21	411	0.04
Power	6.8	10.24	412	0.04
Threshold	8.4	10.09	411	0.06

Table 3: DIAM 2.3 results for 2020 (optimal Abatements, Emissions, Concentrations, Cost) for each impact function. In the linear case, uncertainty is about the scale of a linear function. In the power case, uncertainty pertains to the exponent of the impact function. And in the threshold case, the uncertainty is on the level of the CO_2 threshold.

While there is only one third of additional emission reduction with the threshold damage function, reduction costs are about one half larger as they are in the other cases. Numerical differences are magnified when moving from abatement levels to reduction costs. This is because faster reductions lead to more than proportionally higher costs, an idea that DIAM is designed to model with a high inertia of the energy systems. On the other hand, the dynamics of the carbon cycle implies that over the next decades the carbon dioxide concentration is very insensitive to policy actions.

In the threshold case the non linearity threshold is only attained in the high change state of the world, at $Z^H = 481$ ppmv. In that case, it is reached as soon as 2050. However, even if non-linear damages appear early, they don't appear to reach very high levels. In these runs, they were never greater than 0.13% of the world GDP, although the ceiling is at 4%. At the time they peak (2100–2110), they represent 14–16% of the total damages and costs.

Note that in the linear and the power case, the damages are different across each state of the world as early as 2020. This is internally inconsistent with the idea that information arrives only in 2040 in the model. In the threshold case, the uncertain non-linear damages are still very low at the date of the resolution of the uncertainty on the H state of the world because the first threshold has not been reached and thus the damages are quasi identical to zero in the three states of the world. This is more consistent with the assumption of unobservability until 2040.

The timing of abatement in the threshold case is also interesting, because some additional abatement is done well before the damages happen. As said above, there is an additional 1.5% of emission reductions in 2020 with regard to the other cases, although the non-linear damage represents only 0.007 of the world GDP in 2070. Thus, this optimal emission path case could be considered as an illustration of a precautionary path: costly additional abatement effort is optimal well before the non-linear damages are even measurable.

The effect of learning on the optimal emission path is illustrated in the table 4. We compare, for each case, the percentage of emission reductions with three possibilities regarding learning. The first possibility is the one already presented with sequential action and learning in 2040 and 2060 (act then learn). The second possibility is a no learning case (expected damages). In the third there is no uncertainty at all, and hence a trajectory for each state of the world (learn then act). It appears that there is a strong

	Optimal abatement of global CO_2 emissions in 2020 in percent					
	Sequential decision (act then learn)	No learning (expected damages)	Perfect information (learn then act)			
Case			state	L	C	H
Linear	7.2	7.3		2.0	6.0	15.8
Power	6.8	7.1		4.0	7.1	8.4
Threshold	8.4	11.1		6.0	6.9	12.7

Table 4: Effects of learning and uncertainty on the abatements in 2020 for three different damage functions (linear, power and threshold cases).

	double threshold	vary slope (γ)			reference
	$d_p \times 2$	0.7	0.5	0.3	0.1
abatement %	8.63	15.20	12.02	9.92	8.45

Table 5: Sensitivity analysis on various parameters of the threshold damage function. The ceiling is doubled and various values for γ, controlling the slope are tested.

effect of learning only in the threshold case.

As discussed above, many previous studies found a very small effect of learning, and had the optimal trajectory with recourse (called 'act then learn') very close to the optimistic full-information trajectory (called 'learn then act'). Our results suggest that one explanation for these results is that they used a power or a linear damage function.

Finally, the sensitivity analysis displayed table 5 on the threshold function parameters shows that the abruptness of the kink does matter: in this model a sharper kink means less effort. This result may be explained simply: when the step is less abrupt, the concentration threshold is also lower because damages start earlier.

On the contrary results are relatively insensitive to the ceiling height, that is the size of the loss on the other side of the nonlinearity. This is because on these optimal trajectories, the worst does not happen. Indeed the results show that as long as the slope isn't too flat the nonlinear region is avoided if possible. This explains why the location of the concentration threshold is the most important parameter of the model. While this is purely a cost-benefit model, that parameter acts as a soft ceiling that limits CO_2 concentration.

5 Conclusion

This paper has revisited the question of the optimal timing of climate policy using a damage function that increases abruptly. This kind of cost-benefit analysis avoids some fundamental problems of cost-efficiency analysis: there is no a priori environmental constraint that must be met at all cost.

In most of the existing literature, J-shaped hockey-stick or power damage functions were used. A common result of the EMF-14 study was that, compared to the expected damage, sequential decision-making justified only a very small amount of additional precaution. In this paper, we used an S-shaped damage function. This is more consistent with the stylized belief that climate damages are expected to remain small. We find that this model of abrupt change justifies a larger amount of precaution in sequential decision making.

We conclude that the introduction of thresholds, and the uncertainty about the value of the threshold in unfavourable cases appears important for the decision, while introducing non-linearities with the exponent of the damage function do not change the timing of the action. With threshold damage functions and information about the bad case arriving in 2040, it is optimal to reduce emissions well before the threshold is attained, and also before the damages happen.

We argue that realistic parameters of the S-shaped damage function g are easier to know than parameters of the power law damage function f. This is because the critical parameter (the location of the dangerous CO_2 concentration threshold) can be related to geophysical knowledge about the climate system. Statistics based on climate simulation model results could be used to calibrate uncertainty on that. Regarding the damage function $D = \theta_1(\Delta T)^{\theta_2}$, it seems comparatively harder to avoid subjective assessments when the uncertain parameters are the scale θ_1 or the exponent θ_2.

Admittedly, the S-shaped damage function also takes a scale parameter d_p that is as hard to know as θ_1. But uncertainty about d_p seems less critical than uncertainty about θ_1. We found the optimal trajectory to be much more sensitive to the location of the threshold than to the magnitude of the loss.

With an S-shaped function, marginal damages can increase rapidly and this acts as a soft ceiling on carbon dioxide concentration in these simulations. We found that the possibility of a relatively low loss of GDP of 4%, if happening early and abruptly can justify some additional efforts of mitigation in the near term. This is in agreement with recent results by Keller et al. [2004], showing that a suprisingly small threshold specific damage (about 0.5%) significantly increases the optimal CO_2 abatement.

With the representation of non-linearity and uncertainty presented here, a kind of precautionary behaviour is revealed by the cost-benefit analysis of optimal reduction paths: we can not wait for damages to happen before mitigating more. This result became only visible when the model explicitly integrated an uncertain threshold. Accounting for the possibility of abrupt and near term climate change is crucial to properly understand climate policy.

References

Wallace Broecker. Thermohaline circulation, the Achille's heel of our climate system: Will man-made co_2 upset the current balance. *Science*, 278(5343):1582–1588, 1997.

Michael G. Dalton. The welfare bias fom omitting climatic variability in economic studies of global warming. *Journal of Environmental Economics and Management*, 33:221–239, 1997.

Avinash K. Dixit and Robert S. Pindyck. *The timing of environmental policy*, chapter 12.3, pages 412–418. Princeton University Press, Princeton, New Jersey, 1994.

Jon Gjerde, Sverre Grepperud, and Snorre Kverndokk. Optimal climate policy under the possibility of a catastrophe. *Resource and Energy Economics*, 21:289–317, 1999.

Minh Ha-Duong. *Comment tenir compte de l'irréversibilité dans l'évaluation intégrée du changement climatique ?* Thèse de doctorat, École des hautes Études en Sciences Sociales, Paris, April 1998. URL `http://www.centre-cired.fr`.

Minh Ha-Duong, Michael J. Grubb, and Jean-Charles Hourcade. Influence of socioeconomic inertia and uncertainty on optimal CO_2-emission abatement. *Nature*, 390:270–274, 1997. URL `file://HaDuong.ea-1997-InfluenceInertiaUncertaintyAbatement.pdf`.

IPCC. *Climate Change 2001: Synthesis Report. A contribution of Working Groups I, II, and III to the Third Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press, 2001. ISBN 0-521-01507-3. URL `http://www.ipcc.ch`.

Klaus Keller, Benjamin M. Bolker, and David F. Bradford. Uncertain climate tresholds and economic optimal growth. *Journal of Environmental Economics and Management*, 48(1):723–741, July 2004.

Alan S. Manne. Hedging strategies for global carbon dioxide abatement.. A summary of poll results. Document de travail du EMF 14 Subgroup – Analysis for Decisions under Uncertainty, April 1996.

Urvashi Narain and Anthony C. Fisher. Irreversibility and catastrophic global warming. In *First World Congress of Environmental and Resource Economists*, Venice, Italy, June 25-27 1998. EAERE, AERE. Deposited electronically at the GNEE archive.

William D. Nordhaus. *Managing the Global Commons*. MIT Press, 1994.

Stephen C. Peck and Thomas J. Teisberg. The importance of nonlinearities in global warming damage costs. In Joel Darmstadter and Michael A. Toman, editors, *Assessing Surprises and Nonlinearities in Greenhouse Warming*. Ressources For the Future, May 1993.

Gary Yohe and Rodney Wallace. Near term mitigation policy for global change under uncertainty: minimizing expected cost of meeting unknown concentration tresholds. *Environmental Modeling and Assessment*, 1:47–57, 1996.