IIASA "Learning About Climate Change" conference
Laxenburg, 10–11 April 2006

Learning as combining imprecise evidence

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Alternative models of beliefs suit alternative questions

1. Classical, precise probabilities: Detecting climate change trend
2. Imprecise probabilities: Inductive inference for rare events
3. Information fusion (Dempster-Shafer): social construction of belief from experts' opinions

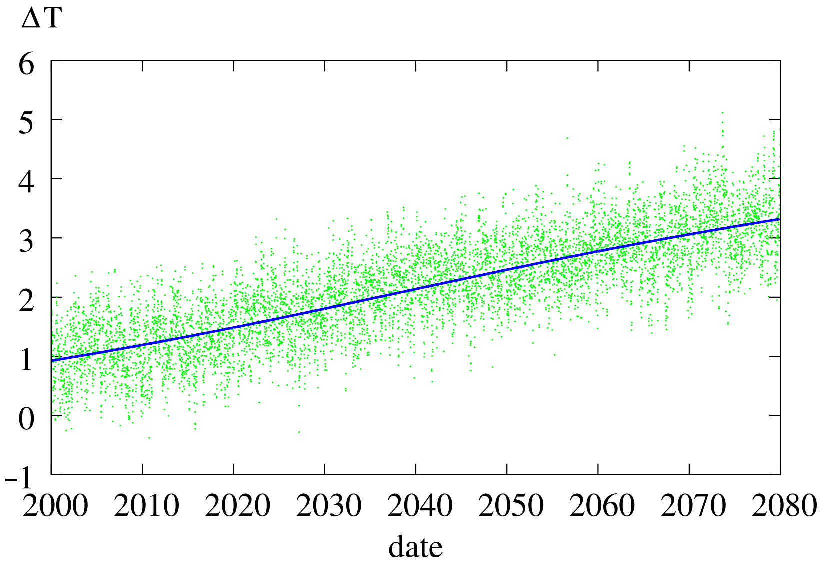
1. Classical, precise probabilities: Detecting climate change trend

Is anticipation of climatic change important ?

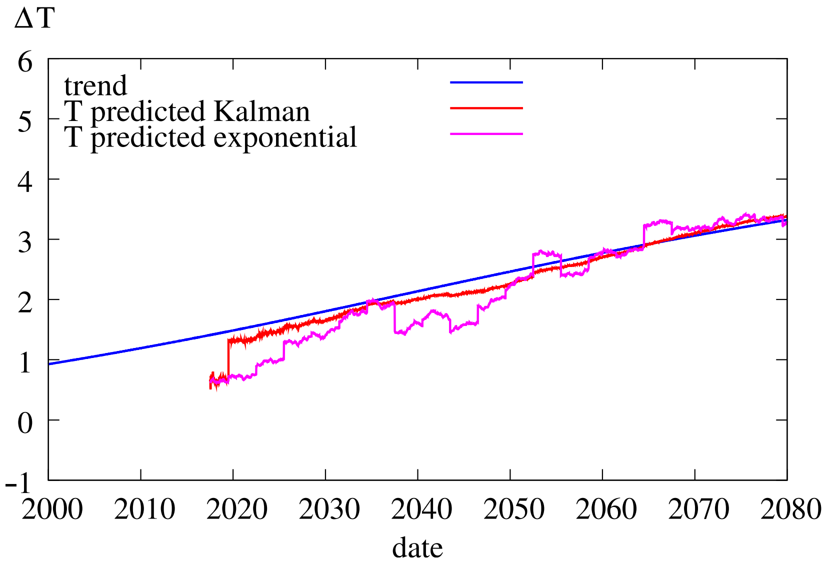
Regarding weather-related infrastructures, assumed to last $\sim$55 years, compare three investment rules:

1. Reactive adaptation. Investment designed for current temperature.
2. Simple proactive adaptation. Investment designed for predicted temperature at capital mid-life. No model, exponential forgetting for temperature and its trend.
3. Sophisticated proactive adaptation: Linear model with a Kalman filter to detect climate sensitivity.

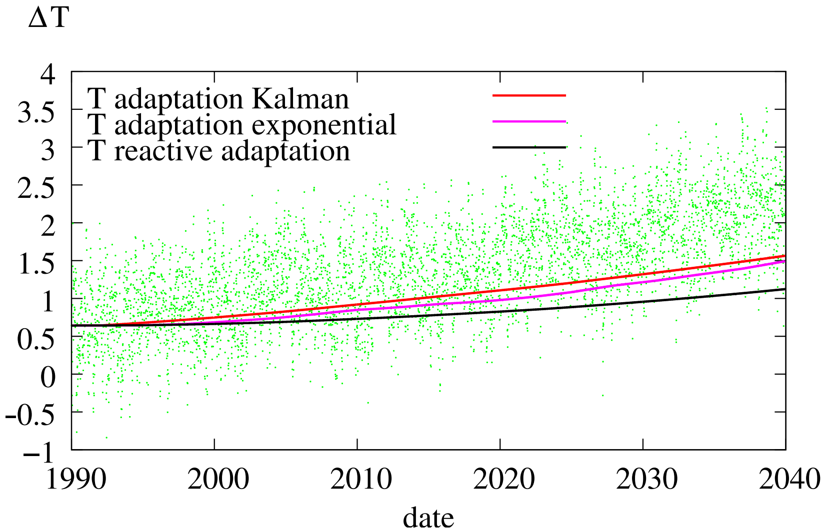
(Unknown) Trend and variability



Temperature prediction



Evolution of the capital stock average design temperature



Discussion of 1. Climatic change anticipation

- Without learning, we are out of the natural variability range by mid-century
- Proactive learning makes a difference, even with the simple rule
- What about model uncertainty ?

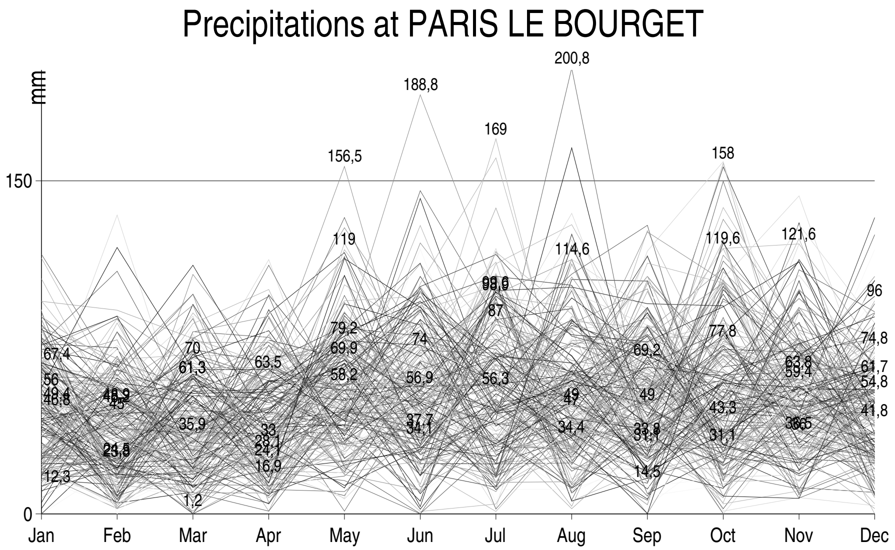
2. Inference for rare events with imprecise probabilities

Probability of a wet month in Paris next year?

An imprecise answer is a probability range : 2.5 to 5 percent. It may be better justified than a precise number:

- Trade off between precision and confidence (back of the enveloppe calculation)
- When subjective priors inadequate, information imprecise, data missing
- Extreme case: possibility of an event that has never been observed.

In 219 years, 9 months over 150mm precipitation



Inductive learning under uncertainty (ambiguity)

The frequency after the next (unknown) observation will be:

More than $\frac{9}{219+1}$ but less than $\frac{9+1}{219+1}$

For m positive outcomes in n trials, imprecise beta model infers:

$$[\frac{m}{n+s}, \frac{m+s}{n+s}] \tag{1}$$

Parameter s determines the degree of imprecision in posterior inferences. It can be interpreted as a number of additional unknown observations.

Mathematical break: the imprecise beta model as robust bayesian inference

Let θ denote the chance of success in a Bernouilli trials experiment. Assume the prior on θ is the familly of PDFs

$$M = \{\beta(s,t),\ 0 < t < 1\} \tag{2}$$

where the beta laws $\beta(s,t)(\theta) \propto \theta^{st-1}(1-\theta)^{s(1-t)-1}$ are parametrized by their mean t. Bayesian updating for m successes in n trials lead to posterior PDFs

$$M' = \left\{\beta(s+n, \frac{st+m}{s+n}),\ 0 < t < 1\right\} \tag{3}$$

The lower probability bound is

$$\inf_{p \in M'} E\theta = \frac{m}{s+n}$$

Google "Imprecise Dirichlet Model" for the multinomial case.

Results: probability of occurence next year (per cent)

Wet month in Paris

Observation period	n	m	s	Result
1870–1989	219	9	0	4.1 sharp
	219	9	1	4.1 – 4.5
	219	9	2	4.1 – 5.0
1900–	89	3	1	3.3 – 4.4
1950–	39	1	1	2.5 – 5.0

Major nuclear accident

Observation period	n	m	s	Result
1950–2006	56	2	1	3.5 – 5.3
1986–2006	20	0	1	0 – 4.8

Discussion of 2: inference with imprecise probabilities

- A robust bayesian approach, imprecision meaningful when s/n is not negligible.
- Far-reaching consequences: decision making with imprecise expected utility, logic
- Some empirical evidence for expected value as an intervall
- Events never or rarely observed: maximum probability = degree of possibility

[3.] Learning in the Transferable Belief Model: Fusion of experts opinion

Possibility distribution of climate sensitivity $\Delta T(2 \times CO_2)$?

'Evidence' to learn from:
Morgan and Keith (1995) experts elicitation survey.

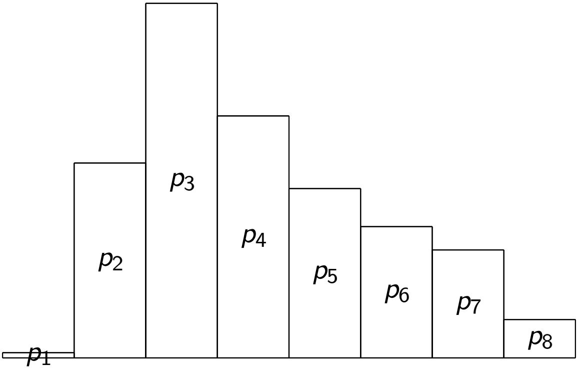
Problems:

- Information given as probability distributions functions
- Experts are not independant and not equally trusted
- Conflicting opinions

Probability distribution from expert 1

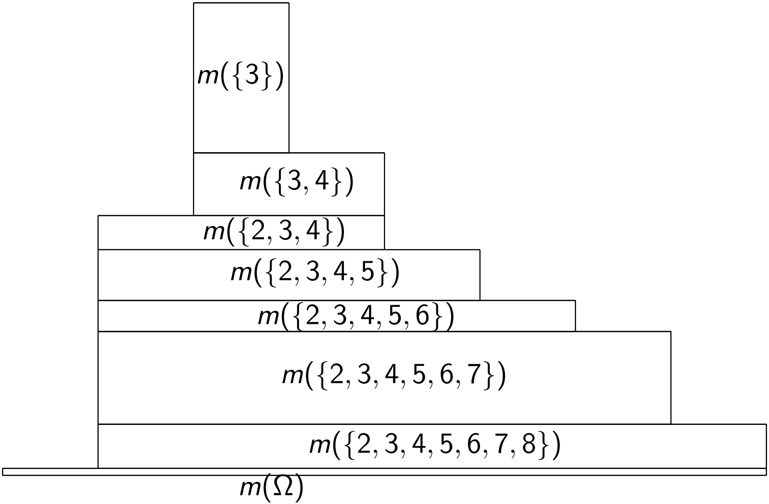
$$p_1 = P\left(-6 \le \Delta T_{2\times CO_2} \le 0\right)$$

$$p_2 = P\left(0 \le \Delta T_{2\times CO_2} \le 1.7\right) \ \ldots$$

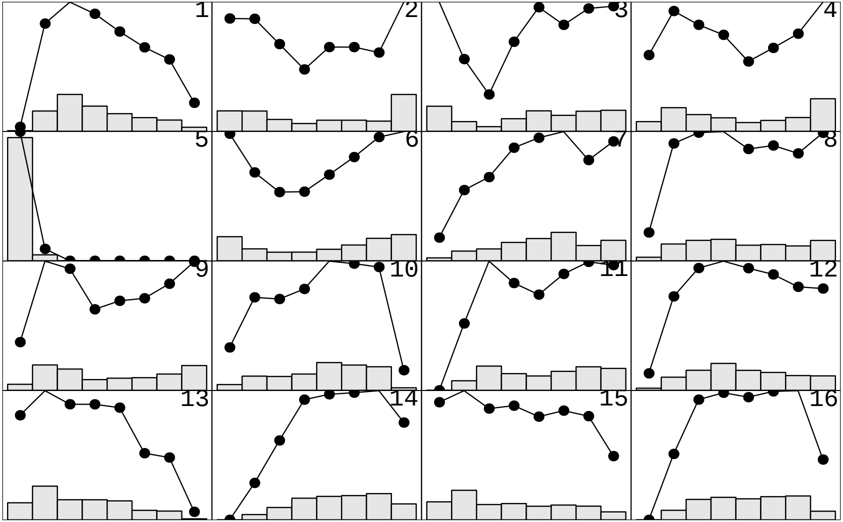


Belief function from expert 1

Define the possibility function : $\pi(\omega) = \sum_{\omega \in A} m(A)$



Results from 16 experts: find the outlier!



Dotted lines: possibility, grey histograms: probability

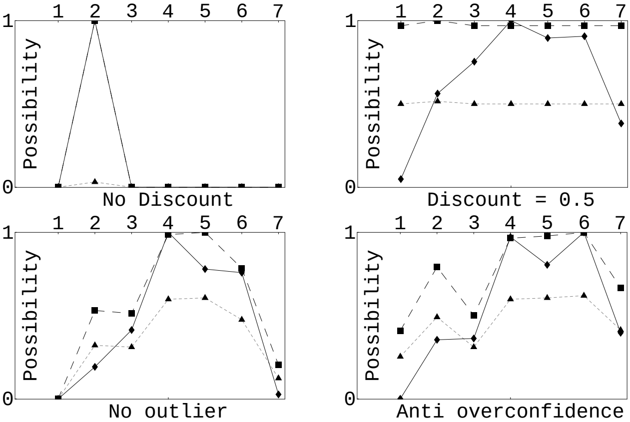
Fusion in two stages

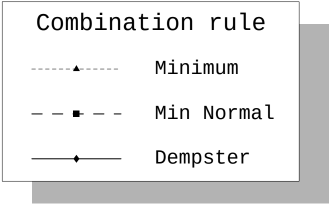
1. Ignoring the outlier 5, and pool other expert's belief functions using a conjunction operator that do not assume independance (idempotent).

2. To combine these prior beliefs with expert 5 beliefs. To learn from new evidence, one need to model the relation with the prior : Discount outlier's opinion and how much ? Independance ? Logical connection ?

- Conjunction: Pool AND 5 are true
- Disjunction: Pool OR 5 is true
- Exclusive Disjunction: Either pool XOR 5 is true, not both

Conjunction: pool AND discounted(5) are right

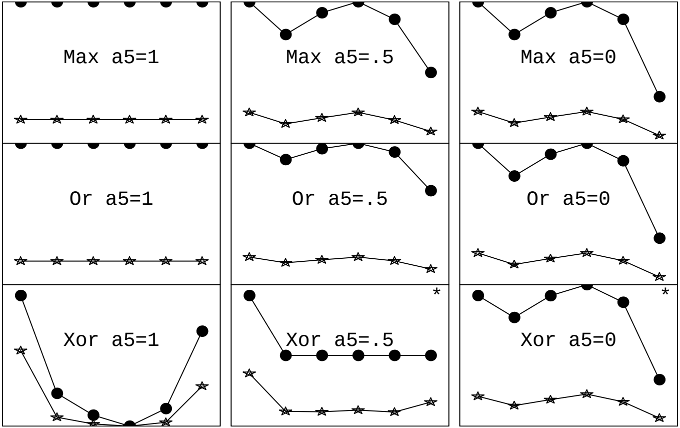




Bin	Climate sensitivity
1	-6.0 to 0.9 °C
2	0.9 to 1.7 °C
3	1.7 to 2.2 °C
4	2.2 to 2.7 °C
5	2.7 to 3.3 °C
6	3.3 to 4.2 °C
7	4.2 to 12.0 °C

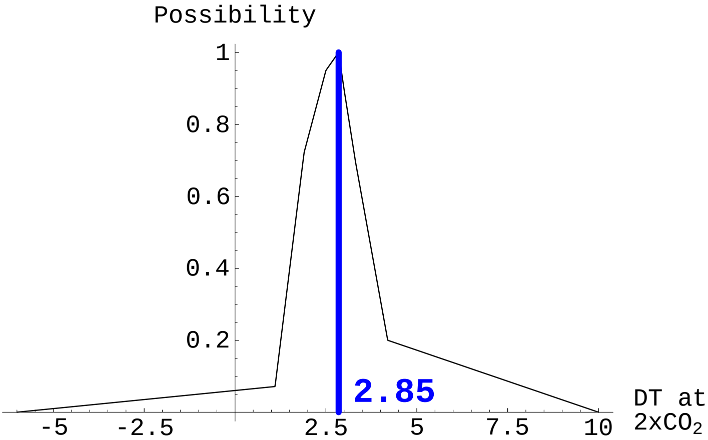
Disjunction: OR (first two rows), XOR bottom row

Left col: $a5 = 1$ means consider outlier's beliefs void (full discount)
Right col: $a5 = 0$ means consider them fully.



Subjective assessment of $\Delta T(2\times CO_2)$

Conjunction, discounting outlier 90 percent.



Discussion of 3. Fusion of experts opinion

- Learning from conflicting informations requires explicit modeling of evidence reliability and sources interactivity
- Defeasible reasoning: X XOR TRUE $= \sim X$
- Formalization of social construction of belief

Conclusion: approaches to formalize learning

- Dectecting climate change matters
- Inference for rare events: imprecise models are robust
- Construction of belief: learning as combining evidence, sometimes conflicting