IPMU'08, June 22-27, 2008, Málaga, Spain

A hierarchical fusion of expert opinion in the Transferable Belief Model (TBM)

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The frame of reference: climate sensitivity

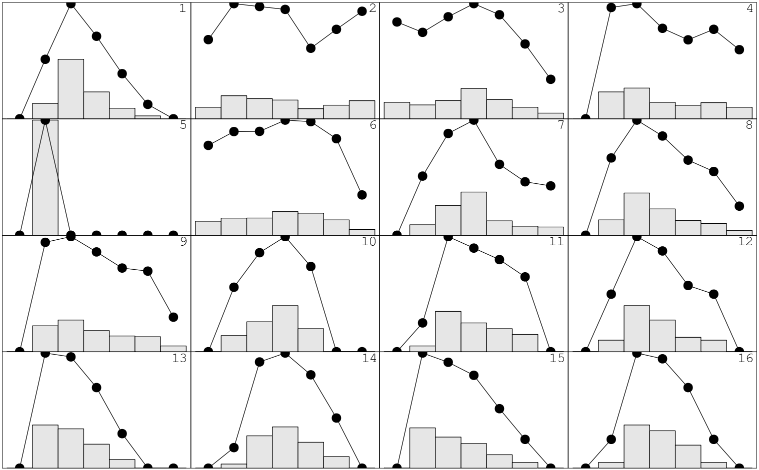
Climate sensitivity $\Delta T_{2\times}$ is:
The long term global warming if $[CO_2]$ in the atmosphere doubles
Uncertain: 1.5°C to 4.5°C.

Morgan and Keith (1995) obtained probability density functions by interviewing 16 leading U.S. climate scientists.

Experts' uncertainty range subdivided in 7 intervalls to simplify:

$$\begin{aligned} \Omega &= \{\omega_1, \ldots, \omega_7\} \\ &= \{[-6, 0],\ [0, 1.5],\ [1.5, 2.5],\ [2.5, 3.5],\ [3.5, 4.5],\ [4.5, 6],\ [6, 12]\} \end{aligned}$$

Variety of views: everything possible {2,3...}, no cooling {4...}, reasonable middle {1...}, no problem {5}



Fusion issues using experts as information sources

- Dependance $\rightarrow$ Avoid unjustified accuracy
- Complete contradiction $\rightarrow$ Need paraconsistency
- Scientific validity $\neq$ popularity $\rightarrow$ No majority rule
- Calibrating experts is not practical $\rightarrow$ don't !

Categorical beliefs: the indicator function $\mathbf{1}_E$

Belief that the state of the world is in the subset $E = \{\omega_2, \omega_3, \omega_4\}$

of the frame of reference $\Omega = \{\omega_1, \ldots, \omega_7\}$ is represented by
$m = \mathbf{1}_E$
the indicator function of E:

$$\begin{cases} m(\{\omega_2, \omega_3, \omega_4\}) = m(E) = 1 & \\ m(A) = 0 & \text{for any other } A \subset \Omega,\ A \neq E \end{cases} \tag{1}$$

Representing belief with a random subset of Ω

We allocate the unit "mass of belief" among subsets of Ω.

$m : 2^\Omega \rightarrow [0, 1]$ is a Basic Belief Assignment iff:

$$\sum_{A \subset \Omega} m(A) = 1 \qquad (2)$$

Corner cases included: ignorance and contradiction

Total ignorance, no information Void beliefs represented by $\mathbf{1}_\Omega$.

Total confusion Contradictory beliefs represented by $\mathbf{1}_\emptyset$.

Discounting and simple beliefs

Discounting is adding a degree of doubt r to a belief m
by mixing it with the void beliefs:

$$\mathrm{disc}(m, r) = (1 - r)\, m + r\, \mathbf{1}_\Omega \tag{3}$$

Denote A^s the simple belief that
"The state of the world is in A, with a degree of confidence s":

$$A^s = \mathrm{disc}(\mathbf{1}_A, e^{-s}) \tag{4}$$

That is:

$$\begin{cases} A^s(A) = 1 - e^{-s} & \\ A^s(\Omega) = e^{-s} & \\ A^s(X) = 0 & \text{if } X \neq A \text{ and } X \neq \Omega \end{cases}$$

Conjunction $\circledcap$ and disjunction $\circledcup$ of beliefs

When two reliable information sources say one A and the other B, believe in the intersection of opinions (TBM allows $\mathbf{1}_\emptyset$):

$$\mathbf{1}_A \circledcap \mathbf{1}_B = \mathbf{1}_{A \cap B}$$

Generally:

$$(\mu_1 \circledcap \mu_2)(A) = \sum_{B \cap C = A} \mu_1(B)\mu_2(C) \tag{5}$$

When at least one source is reliable, consider the union of opinions.

$$(\mu_1 \circledcup \mu_2)(A) = \sum_{B \cup C = A} \mu_1(B)\mu_2(C) \tag{6}$$

Canonical decomposition in simple beliefs

For any m such that $m(\Omega) > 0$, there are weights $\big(s(A)\big)_{A \subsetneq \Omega}$ such that:

$$m = \bigcircledcap_{A \subsetneq \Omega} A^{s(A)} \tag{7}$$

Weights of the $\circledcap$ conjonction are the sum of weights:

$$m_1 \circledcap m_2 = \bigcircledcap_{A \subsetneq \Omega} A^{s_1(A)+s_2(A)} \tag{8}$$

$\bigcircledcap$ Conjunction increases confidence: $A^s \circledcap A^s = A^{2s}$.

Good for independent information sources,
but for experts we want to avoid unjustified accuracy

T. Denœux's cautious combination operator

Whenever...
Expert 1 has confidence $s_1(A)$ that state of the world is in A
Expert 2 has confidence $s_2(A)$
...follow the most confident:

$$m_1 \owedge m_2 = \bigcircledcap_{A \subsetneq \Omega} A^{\max(s_1(A), s_2(A))} \tag{9}$$

Distributivity: $(m_1 \circledcap m_3) \owedge (m_2 \circledcap m_3) = (m_1 \owedge m_2) \circledcap m_3$
Interpretation:
Expert 1 has beliefs $m_1 \circledcap m_3$
Expert 2 has beliefs $m_2 \circledcap m_3$
$\bigcirc\!\!\!\!\wedge$ cautious combination of experts counts evidence m_1 only once.

Historical operators: Averaging and Dempster's rule

Averaging is $\frac{m_1(X)+m_2(X)}{2}$

Renormalizing m means replacing it with m^* such that $m^*(\emptyset) = 0$ and

$$m^*(X) = \frac{m(X)}{1 - m(\emptyset)}$$

Dempster's rule is renormalized conjunction:

$$m_1 \oplus m_2 = (m_1 \circledcirc m_2)^* \qquad (10)$$

There is no satisfying fusion operator

	Average	$\bigoplus$, $\textcircled{\cap}$	$\textcircled{\wedge}$	$\textcircled{\cup}$
Majority rule	☹	✓	✓	✓
Contradiction	✓	☹	☹	✓
Unjust. accuracy	✓	☹	✓	☹

Discounting decreases contradiction issues,
but calibrating experts is not practical.

A hierarchical approach

1. Partition experts in schools of thought (adaptative or sociological methods)
2. Within groups, $\textcircled{\wedge}$ cautious combination
3. Across theories, $\textcircled{\cup}$ disjonction

Using the climate experts dataset:

$$
\begin{array}{rcll}
m_A & = & m_2 \textcircled{\wedge} m_3 \textcircled{\wedge} m_6 & \text{Everything possible} \\
m_B & = & m_4 \textcircled{\wedge} m_7 \textcircled{\wedge} m_8 \textcircled{\wedge} m_9 & \text{No cooling} \\
m_C & = & m_1 \textcircled{\wedge} m_{10} \textcircled{\wedge} \cdots \textcircled{\wedge} m_{16} & \text{Reasonable middle} \\
m_D & = & m_5 & \text{No problem} \\
m & = & m_A \textcircled{\cup} m_B \textcircled{\cup} m_C \textcircled{\cup} m_D &
\end{array}
$$

Probability and plausibility used to present results

Any m defines a probability p^m by:

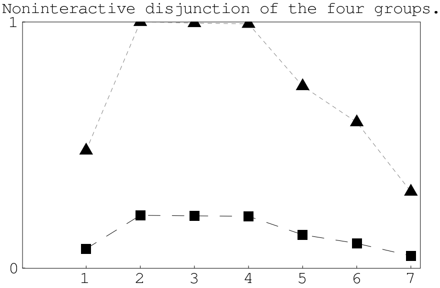
$$p^m(\omega_i) = \sum_{X \ni \omega_i} \frac{m^*(X)}{|X|} \tag{11}$$

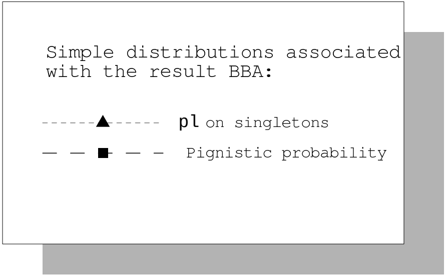
Any m defines a plausibility function pl,
which is given on singletons by:

$$pl(\{\omega_i\}) = \sum_{X \ni \omega_i} m(X) \tag{12}$$

Levels of probability are generally smaller than levels of plausibility.

Results: fusion of 16 experts on $\Delta T_{2\times}$, MK 1995 survey





i	1	2	3	4	5	6	7
ω_i	-6,0	0,1.5	1.5,2.5	2.5,3.5	3.5,4.5	4.5,6.0	6.0,12
pl	0.48	1.	1.	0.99	0.74	0.59	0.31
p^m	0.08	0.21	0.21	0.21	0.14	0.10	0.05

Hierarchical better than symmetric fusion for expert aggregation

	Average	$\oplus$, $\textcircled{$\cap$}$	$\textcircled{$\wedge$}$	$\textcircled{$\cup$}$
Majority rule	☹	✓	✓	✓
Contradiction	✓	☹	☹	✓
Unjust. accuracy	✓	☹	✓	☹

Fusion method	$m(\Omega)$	$\leq 1.5°$C *bel–pl*	In range *bel–pl*	$\geq 4.5°$C *bel–pl*
Hierarchical	0.18	0.–1.	0.–1.	0.–0.61
Average	0.08	0.07–0.69	0.27–0.93	0.–0.45
disc. Dempster	0.	0.02–0.03	0.97–0.98	0.–0.
Disjunction	0.99	0.–1.	0.–1.	0.–1.

The likelihood of $\Delta T_{2x} < 1.5^\circ\mathrm{C}$ has decreased since 1995

IPCC 2001: Climate sensitivity is likely to be in the 1.5 to 4.5°C range (unchanged from 1979)

$\Delta T_{2x} \in \ldots$	$[0^\circ\mathrm{C}, 1.5^\circ\mathrm{C}]$	$[1.5^\circ\mathrm{C}, 4.5^\circ\mathrm{C}]$	$[4.5^\circ\mathrm{C}, 10^\circ\mathrm{C}]$
Published PDFs	[0, 0.07]	[0.31, 0.98]	[0.02, 0.62]
Kriegler (2005)	[0, 0.00]	[0.53, 0.99]	[0.01, 0.47]

IPCC 2007: [2, 4.5°C] is likely, below 1.5°C is very unlikely.

Note:
Likely means $0.66 \leq p \leq 0.90$,
very unlikely means $p \leq 0.1$.

Conclusions

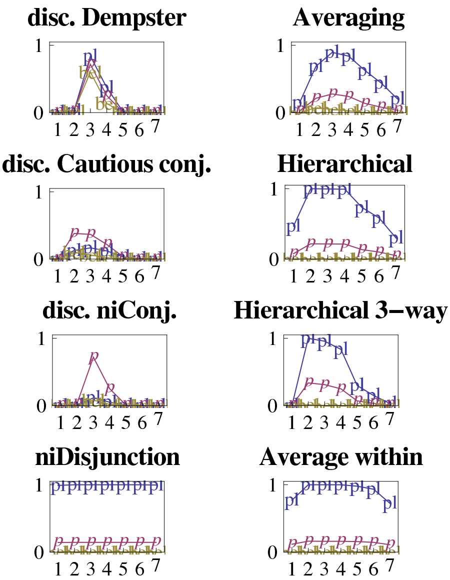
A hierarchical approach to fusion expert opinions:

- Imprecise
- Deals with dependencies and contradiction
- Avoid majority rule and calibration
- Requires a sociological study of experts groups

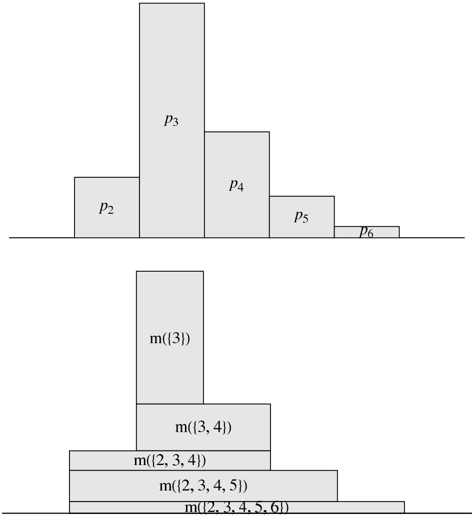
About climate sensitivity:

- Above 4.5°C was already plausible in 1995
- Below 1.5°C is less plausible today

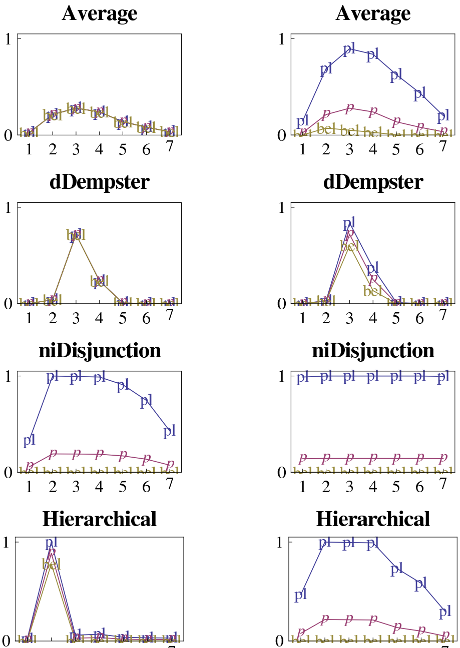
Symmetric fusions operators vs. Hierarchical approaches



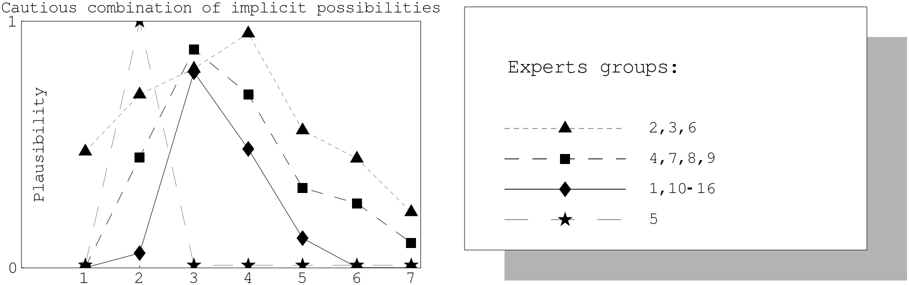
Expert 1: bayesian m (top), consonnant m (bottom)



Sensitivity analysis. Bayesian left, consonnant right.



Cautious combination within groups



Result of the hierarchical fusion: the belief function

subset A	$m^*(A)$
$\{2\}$	0.0001
$\{3, 2\}$	0.0074
$\{4, 2\}$	0.0033
$\{4, 3, 2\}$	0.1587
$\{4, 3, 2, 1\}$	0.0064
$\{5, 4, 2\}$	0.0011
$\{5, 4, 3, 2\}$	0.1321
$\{5, 4, 3, 2, 1\}$	0.0709
$\{6, 4, 3, 2\}$	0.0267
$\{6, 4, 3, 2, 1\}$	0.0129
$\{6, 5, 4, 3, 2\}$	0.0888

subset A (cont.)	$m^*(A)$
$\{6, 5, 4, 3, 2, 1\}$	0.1811
$\{7, 4, 3, 2\}$	0.0211
$\{7, 5, 4, 3, 2\}$	0.0063
$\{7, 6, 4, 3, 2\}$	0.0135
$\{7, 6, 4, 3, 2, 1\}$	0.0105
$\{7, 6, 5, 4, 3, 2\}$	0.0632
$\{7, 6, 5, 4, 3, 2, 1\}$	0.1956