Scenarios, probability and possible futures

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Minh Ha-Duong

HaDuong@centre-cired.fr

CIRED, CNRS

1. Outline

1. Intro on scenarios and probabilities.
2. Defining possibility.
3. How (im)plausible should scenarios be ?

Any precise scenario probability is 0

IPCC wrote about +5.5° C in 2100

- SRES: No preferred scenario, no probabilities
- Commonsense: plausibility levels vary

An everlasting controversy:

- Probabilistic Forecasts: $\{F_i, p_i\}$
- Scenarios without quantified belief: $\{S_i\}$

Possibility theory agrees with both SRES team and commonsense: $\{S_i, \pi_i\}$
Fuziness in the F_i is another topic

Social sciences beyond ambiguity

Scenarios are also used for:

- Surprises
- Taboos
- Values
- Strategic uncertainty

2. Defining possibility (informally)

π = 1 - degree of surprise   Shackle (1953)

A subjective function related to beliefs about an event X (Zadeh 1978, Dubois et Prade 1988)

- X is impossible: $\pi(X) = 0$
- X is perfectly possible: $\pi(X) = 1$

Normalisation and maxitivity axioms

π(less surprising future) = 1

If A and B are two future events
with possibility levels $\pi(A)$ and $\pi(B)$,
then possibility of 'A ou B' is the maximum of the two.

Formal definition

Possibility distribution: a function $\pi(x)$ defined for any $x \in \Omega$ into [0, 1], such that its maximum is 1.

Having π on the singletons, we can define the possibility of any subset $A \subset \Omega$ with:

$$\pi(A) = \max_{\omega \in A} \pi(\omega)$$

Which indeed verifies:

$$\pi(A \ \cup \ B) = \max(\pi(A), \pi(B))$$

even if they overlap.

Possibility as imprecise probability

π defines a set of admissible probability distributions $\mathcal{C}$:

$$p \in \mathcal{C} \Longleftrightarrow p(A) \leq \pi(A) \text{ for all } A \subset \Omega$$

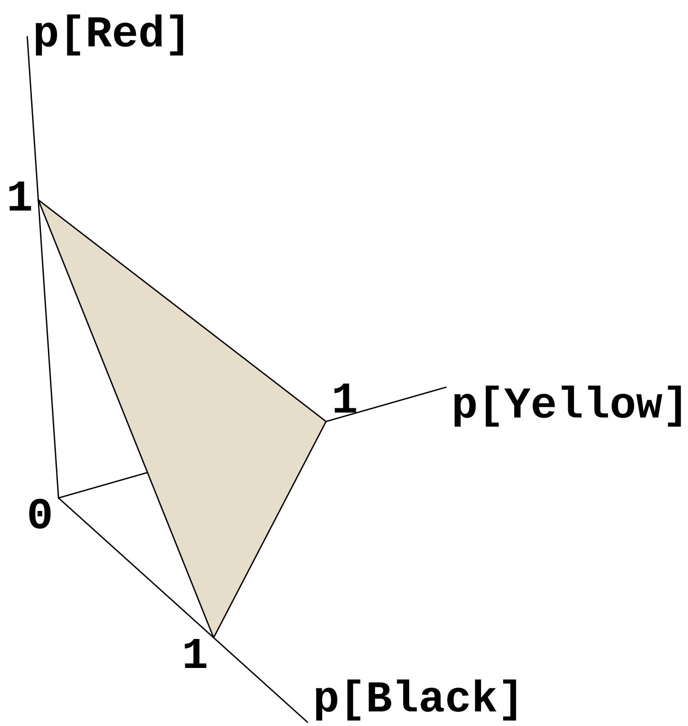
Saying that the possibility of A is $\pi(A)$ amounts to say that the probability of A is smaller than $\pi(A)$.

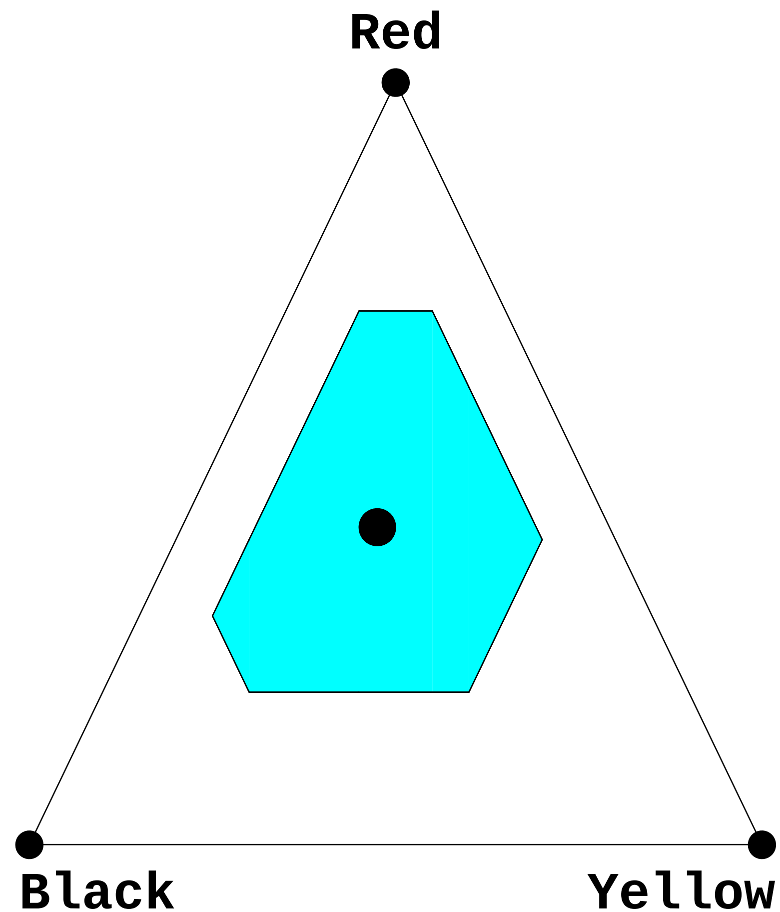
De Finetti view: bet that A will not happen if and only if it pays more than

$$\frac{\pi}{1-\pi} : 1$$

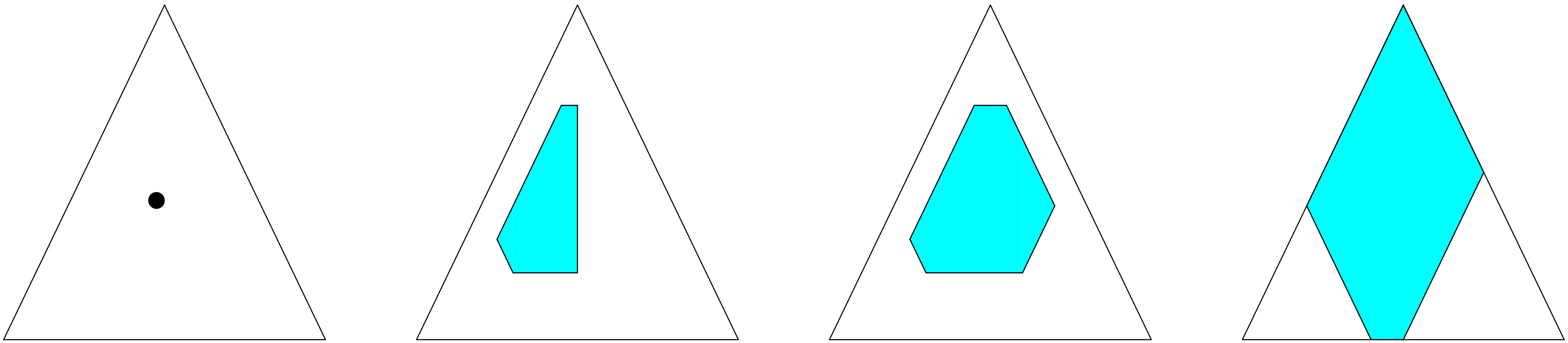
The space of Ellsberg's urns

Each point in the triangle represents a probability distribution.
Blue: a set of admissible probability distributions $\mathcal{C}$





More or less imprecise probabilities



These sets represent increasingly ambiguous beliefs, from precise probabilistic (left) to possibilistic (right).

3. How plausible should scenarios be ?

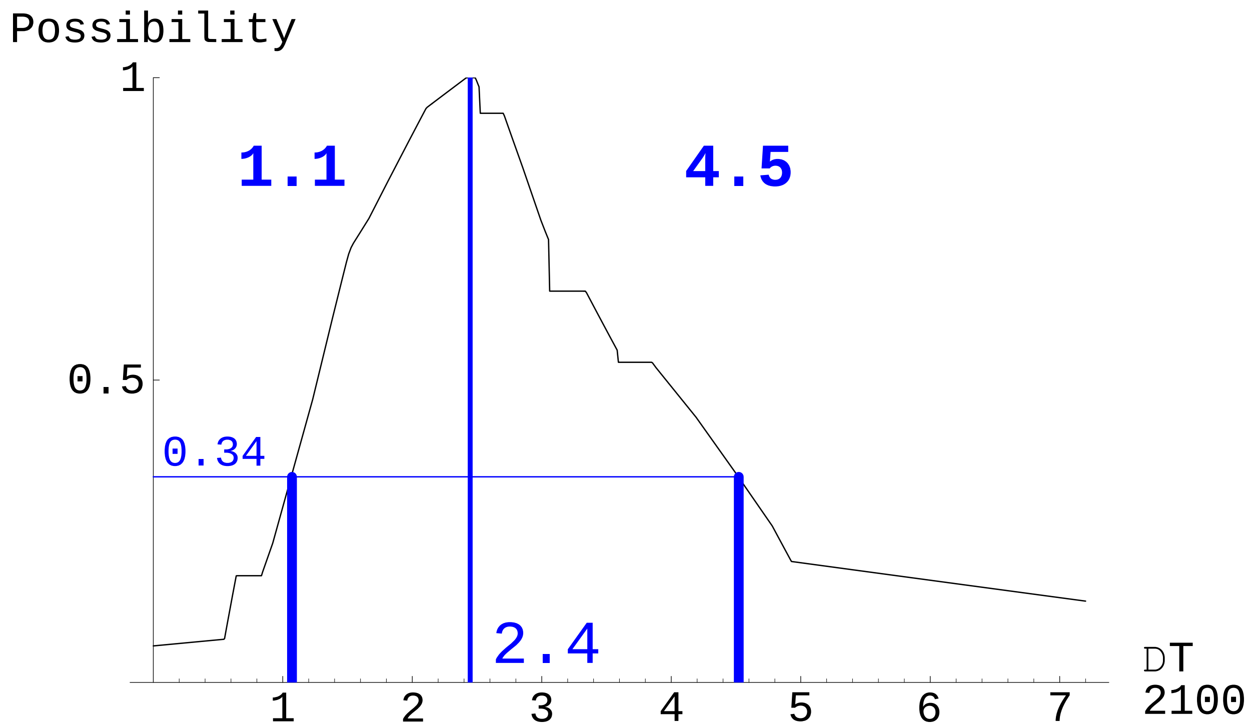
Assuming we have:

- Frame of reference: states of the world which can be described Ω.
- Goals and values: an objective function J (e.g. global warming).
- Ambiguous knowledge: multiple priors $\mathcal{C}$.

We propose a principled method to determine a small number of plausible futures $\{S_i, \pi_i\}$

Example result

A set of three scenarios for global warming in 2100. One at π = 1, the other two at $\pi = \frac{1}{3}$.



So +5.5° C seems unlikely (<20%).

Scenarios making principles

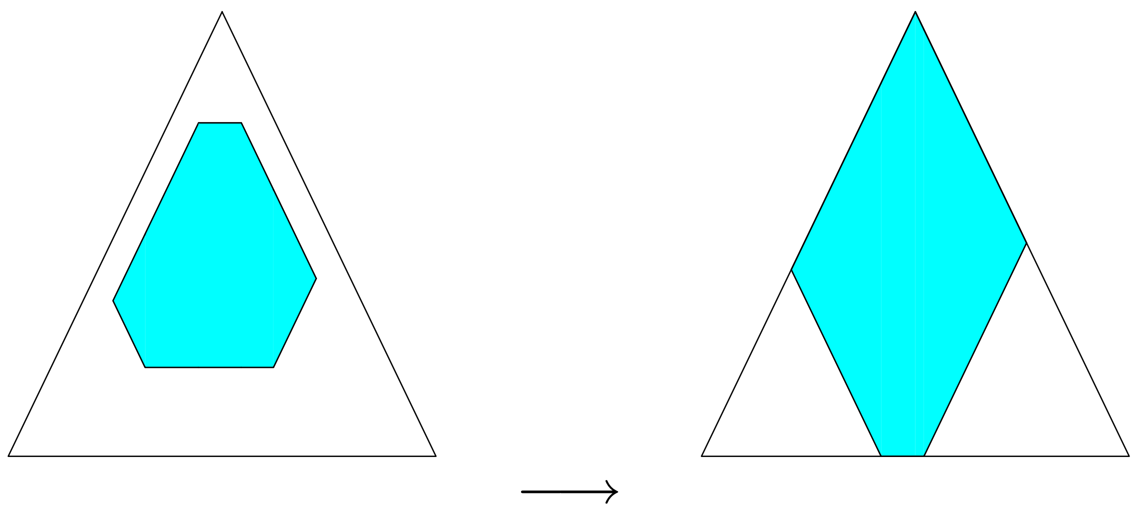
Summarize available information $\mathcal{C}$ by assessing the possibility distribution of the objective J, then choose scenarios S according to multiple criteria:

- Do not restrict possibilities
- Include a perfectly possible scenario
- Keep equiprobability admissible
- Contrast extremes

P1: Do not restrict possibilities

If the expert believes something is possible, it should say so:

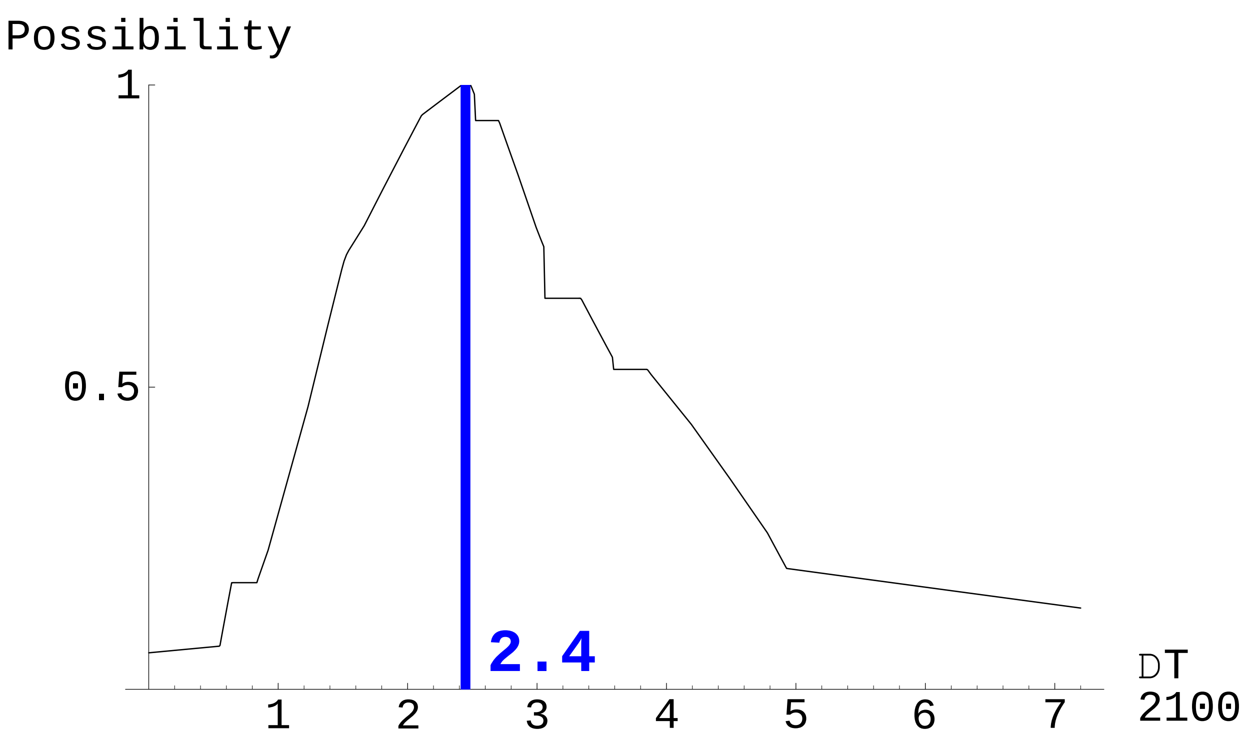
$$\pi(s) \geq \max_{p \in \mathcal{C}} p(s)$$



Enlarging beliefs reduces the set of desirable gambles.

P2: Include a perfectly possible scenario

- Pros and cons
- If no single business as usual, then include multiple futures at $\pi = 1$



P3: No preferred scenario

Partial ordering defined by $\mathcal{C}$:
A is more probable than B whenever it holds for all admissible probability distributions:

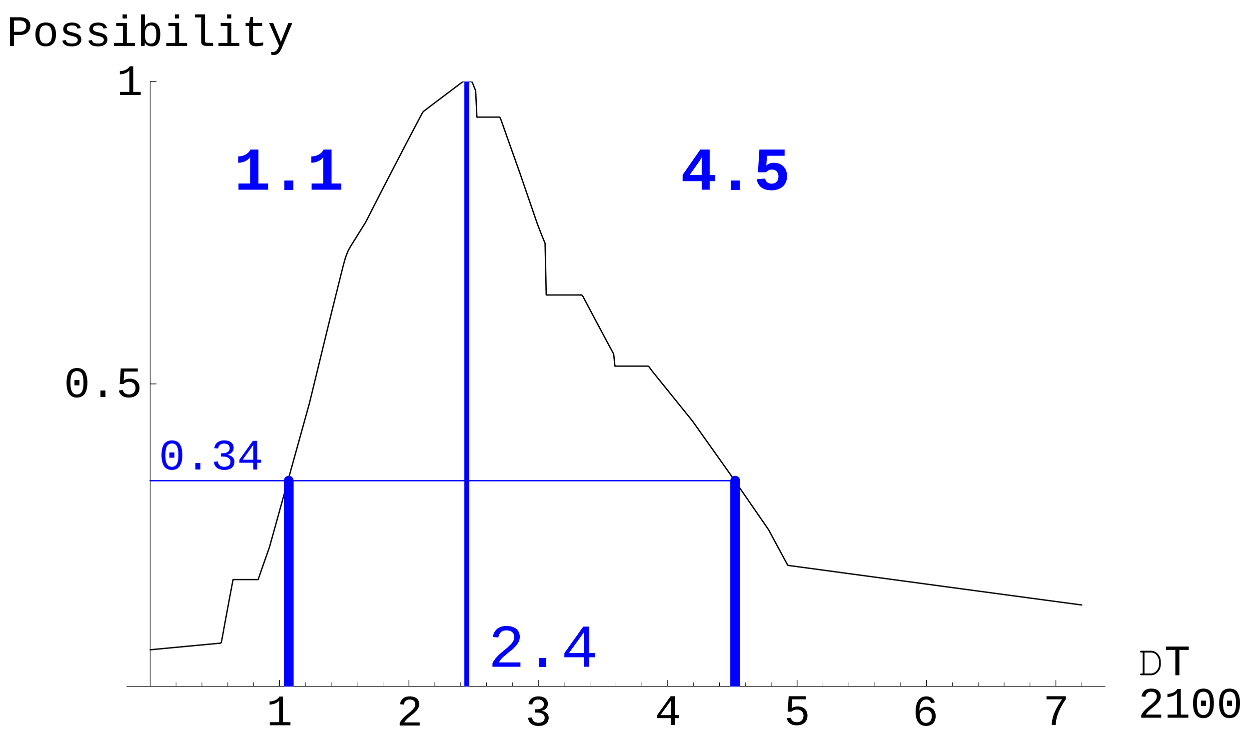
$$\text{for all}\, p \in \mathcal{C},\ p(A) > p(B)$$

It is sufficient to have equiprobability in $\mathcal{C}$ to prevent any preference relation.

$\Rightarrow$ If the expert provides N futures, their possibility should be more than $1/N$.

P4: Contrast extremes

For a given objective function, trade plausibility for extensivity.



Conclusion

This principled scenario-choosing method allows a progressive disclosure of information.

- Less surprising future
- A few number of plausible futures
- Quantified, imprecise beliefs
- The published litterature