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A hierarchical fusion of expert opinion in the Transferable Belief Model (TBM)

Minh Ha-Duong, CNRS, France

Outline

1. Intro: decision and controversies
2. The Transferable Belief Model
3. A hierarchical aggregation procedure

Theoretical teasers:

- No information ($\neq$ equiprobability)
- Contradiction ($\neq$ no information)
- Incompleteness (assimilated to contradiction)
- Negative information

Climate sensitivity $\Delta T_{2\times}$

Long term global warming if $[CO_2]$ in the atmosphere doubles
Uncertain communication anchor: 1.5°C to 4.5°C.

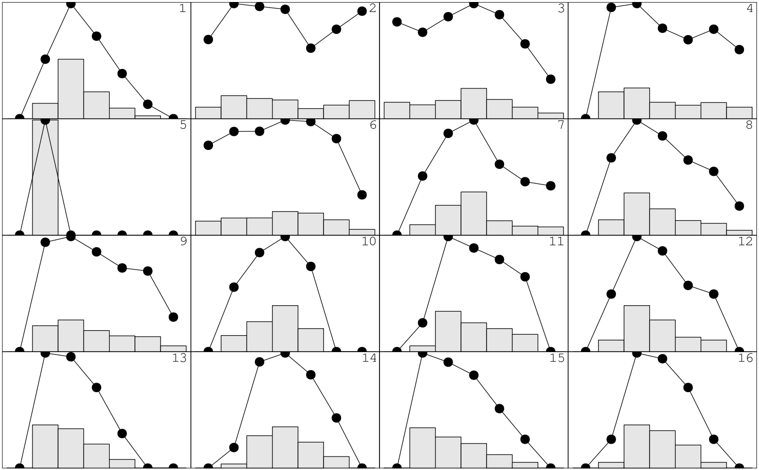
Morgan and Keith (1995) obtained probability density functions by interviewing 16 leading U.S. climate scientists.

Experts' uncertainty range subdivided in 7 intervalls to simplify:

$$\begin{aligned}\Omega &= \{\omega_1, \ldots, \omega_7\} \\ &= \{[-6, 0],\ [0, 1.5],\ [1.5, 2.5],\ [2.5, 3.5],\ [3.5, 4.5],\ [4.5, 6],\ [6, 12]\}\end{aligned}$$

A variety of views

Everything possible {2,3...}, no cooling {4...}, reasonable middle {1...}, no problem {5}



Problems with experts opinions

- Interaction, not independance $\rightarrow$ Avoid unjustified accuracy
- Complete contradiction $\rightarrow$ Need paraconsistency
- Scientific validity $\neq$ popularity $\rightarrow$ No majority rule
- Calibrating experts is not practical $\rightarrow$ don't !

Proposition: hierarchical fusion

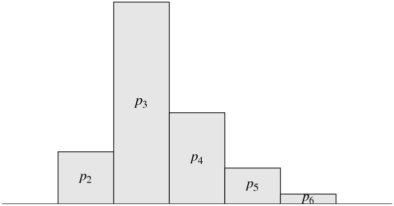
i. Partition experts into groups/school of tought/theories

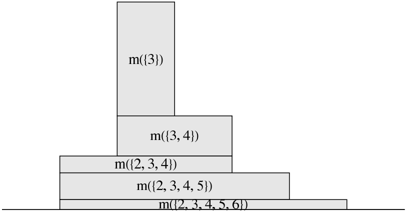
ii. Within each group, cautious combination of opinions

iii. Between groups, disjonction

2. Transferable Beliefs Model

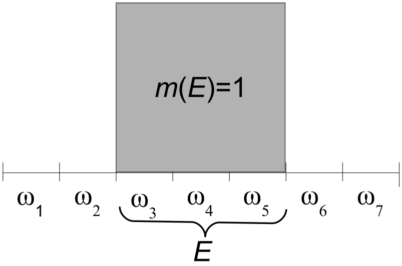
Like Dempster-Shafer, allocate the unit "mass of belief" among subsets of Ω, but allow $m(\{\}) > 0$.





$$m \text{ such that } \sum_{A \subset \Omega} m(A) = 1$$

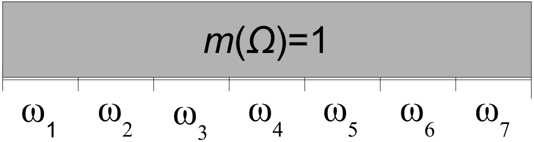
Belief: climate sensitivity is in [1.5,4.5°C]



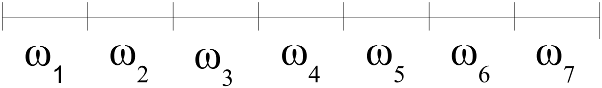
Such categorical beliefs are denoted E^{∞}

Special categorical beliefs

Empty beliefs, no information Ω^{∞}.

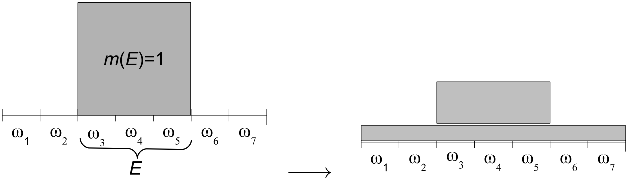


Confusion, too much contradictory d'information $\{\}^{\infty}$.



Doubt, simple beliefs

One can add some doubt to a belief m by diluting it with empty beliefs:



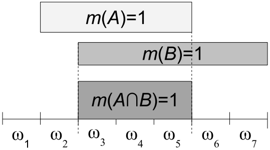
$$\text{doubt}(m, r) = (1 - r)m + r\Omega^{\infty}$$

"The state of the world is E, with a degree of confidence s" is denoted

$$E^s = \text{doubt}(E^{\infty}, e^{-s}) \tag{1}$$

Conjunction ⓒ and disjunction ⓤ of beliefs

When two reliable information sources say one A and the other B, believe in the intersection of opinions (even if empty):



$$A^\infty \circledcap B^\infty = (A \cap B)^\infty$$

More generally (non-normalized Dempster's rule):

$$(\mu_1 \circledcap \mu_2)(A) = \sum_{B \cap C = A} \mu_1(B)\mu_2(C)$$

When at least one source is reliable, consider the union of opinions:

$$(\mu_1 \circledcup \mu_2)(A) = \sum_{B \cup C = A} \mu_1(B)\mu_2(C)$$

Canonical decomposition in simple beliefs

For any m such that $m(\Omega) > 0$, there are weights $\big(s(A)\big)_{A \subsetneq \Omega}$ such that (some weights may be < 0):

$$m = \bigcircledcap_{A \subsetneq \Omega} A^{s(A)} \tag{2}$$

Doing the $\circledcap$ conjonction amounts to adding these weights:

$$m_1 \circledcap m_2 = \bigcircledcap_{A \subsetneq \Omega} A^{s_1(A)+s_2(A)} \tag{3}$$

$\bigcircledcap$ Conjunction increases confidence: $A^s \circledcap A^s = A^{2s}$.

Good for independent information sources,
but unjustified accuracy for interactive experts

T. Denœux's cautious combination operator

Whenever...
Expert 1 has confidence $s_1(A)$ that state of the world is in A
Expert 2 has confidence $s_2(A)$
...follow the most confident:

$$m_1 \owedge m_2 = \bigcircledcap_{A \subsetneq \Omega} A^{\max(s_1(A), s_2(A))} \tag{4}$$

Distributivity: $(m_1 \circledcap m_3) \owedge (m_2 \circledcap m_3) = (m_1 \owedge m_2) \circledcap m_3$
Interpretation:
Expert 1 has beliefs $m_1 \circledcap m_3$
Expert 2 has beliefs $m_2 \circledcap m_3$
$\owedge$ cautious combination of experts counts evidence m_1 only once.

3. All fusion operator are flawed

	Averaging	$\bigoplus$, $\bigcap$	$\bigwedge$	$\bigcup$
Contradiction	✓	☹	☹	✓
False precision	✓	☹	✓	☹
Majority rule	☹	✓	✓	✓

Table: There is no fusion operator that meets the three theoretical challenges.

Adding doubt decreases contradiction, but calibrating experts ?

Hierarachical fusion

i. Partition experts into groups using adaptative methods or sociology)

ii. Within each group, cautious combination of opinions

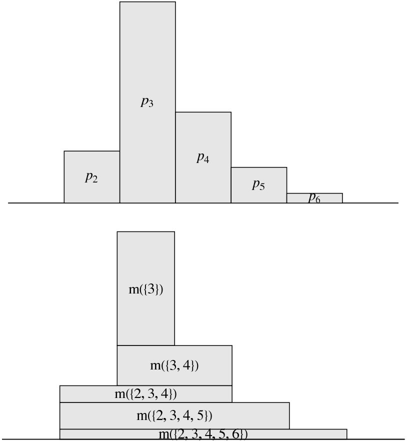
iii. Between groups, disjonction

Using the climate experts dataset:

m_A	$=$	$m_2 \owedge m_3 \owedge m_6$	Everything possible
m_B	$=$	$m_4 \owedge m_7 \owedge m_8 \owedge m_9$	No cooling
m_C	$=$	$m_1 \owedge m_{10} \owedge \cdots \owedge m_{16}$	Reasonable middle
m_D	$=$	m_5	Denial
m	$=$	$m_A \ocup m_B \ocup m_C \ocup m_D$	

How to represent m ?

It spreads an unit mass of belief among the subsets A of Ω



Up to $2^{|\Omega|}$ numbers, where $|\Omega|$ denotes the number of elements of Ω. Inconvenient.

Probability and plausibility

Any m defines a probability p^m by:

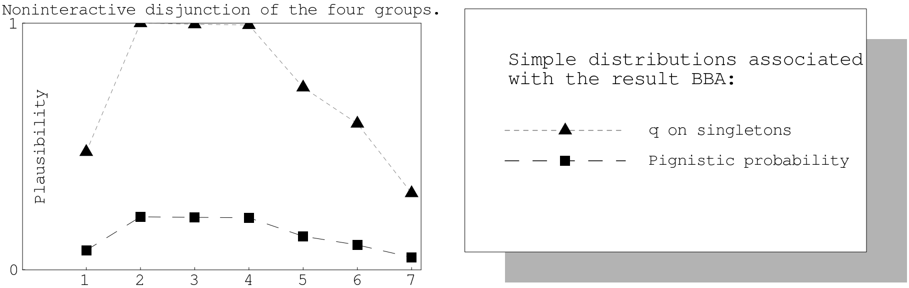
$$p^m(\omega_i) = \sum_{X \ni \omega_i} \frac{m(X)}{|X|} \tag{5}$$

Any m defines a plausibility function pl,
which is given on singletons by:

$$pl(\{\omega_i\}) = \sum_{X \ni \omega_i} m(X) \tag{6}$$

Probability levels are generally less than plausibility levels.

Results: fusion of 16 experts on $\Delta T_{2\times}$, MK 1995



i	1	2	3	4	5	6	7
ω_i °C	-6,0	0,1.5	1.5,2.5	2.5,3.5	3.5,4.5	4.5,6.0	6.0,12
pl	0.48	1.	1.	0.99	0.74	0.59	0.31
p^m	0.08	0.21	0.21	0.21	0.14	0.10	0.05

Belief that $\Delta T_{2x} < 1.5^\circ\mathrm{C}$ decreased since 1995

IPCC then (2001): Climate sensitivity is likely to be in the 1.5 to 4.5°C range (unchanged from 1979).

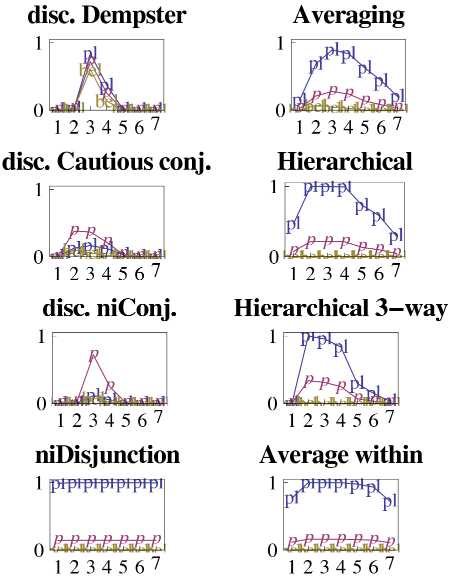
IPCC now (2007): [2, 4.5°C] is likely, below 1.5°C is very unlikely.

$\Delta T_{2x} \in \ldots$	$[0^\circ\mathrm{C}, 1.5^\circ\mathrm{C}]$	$[1.5^\circ\mathrm{C}, 4.5^\circ\mathrm{C}]$	$[4.5^\circ\mathrm{C}, 10^\circ\mathrm{C}]$
Published PDFs	[0, 0.07]	[0.31, 0.98]	[0.02, 0.62]
Kriegler (2005)	[0, 0.00]	[0.53, 0.99]	[0.01, 0.47]

Table: Probability intervalls for climate sensitivity.

Note:
Likely means $0.66 \leq p \leq 0.90$,
very unlikely means $p \leq 0.1$.

Sensitivity analysis to fusion method



Conclusions

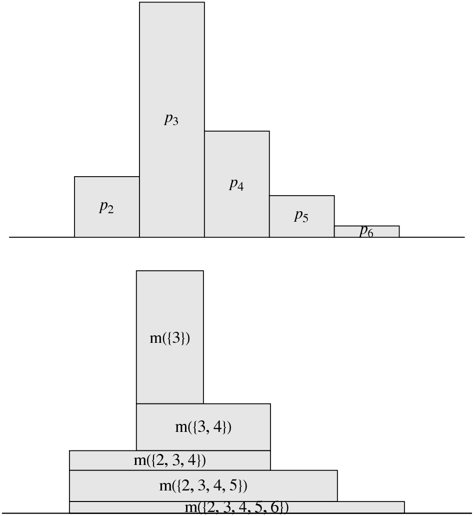
A hierarchical approach to fusion expert opinions:

- Imprecise
- Deals with dependencies and contradiction
- Avoid majority rule and calibration
- Requires a sociological study of experts groups

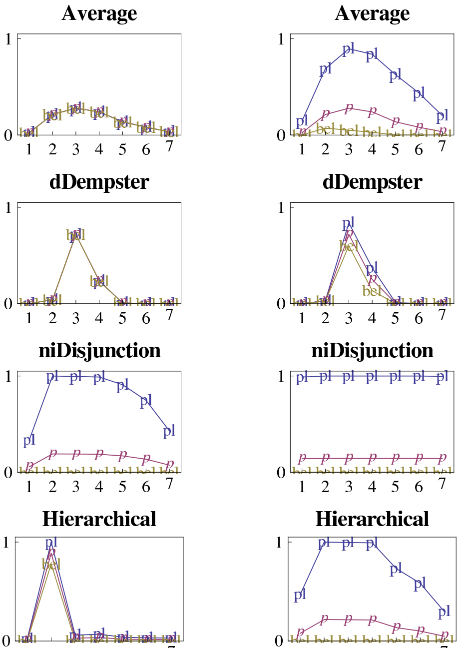
About climate sensitivity:

- Above 4.5°C was already plausible in 1995
- Below 1.5°C is less plausible today

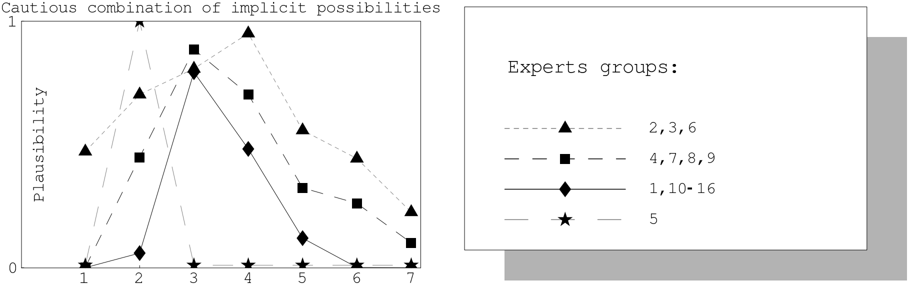
Expert 1: bayesian m (top), consonnant m (bottom)



Sensitivity analysis. Bayesian left, consonnant right.



Cautious combination within groups



Result of the hierarchical fusion: the belief function

subset A	$m^*(A)$
$\{2\}$	0.0001
$\{3, 2\}$	0.0074
$\{4, 2\}$	0.0033
$\{4, 3, 2\}$	0.1587
$\{4, 3, 2, 1\}$	0.0064
$\{5, 4, 2\}$	0.0011
$\{5, 4, 3, 2\}$	0.1321
$\{5, 4, 3, 2, 1\}$	0.0709
$\{6, 4, 3, 2\}$	0.0267
$\{6, 4, 3, 2, 1\}$	0.0129
$\{6, 5, 4, 3, 2\}$	0.0888

subset A (cont.)	$m^*(A)$
$\{6, 5, 4, 3, 2, 1\}$	0.1811
$\{7, 4, 3, 2\}$	0.0211
$\{7, 5, 4, 3, 2\}$	0.0063
$\{7, 6, 4, 3, 2\}$	0.0135
$\{7, 6, 4, 3, 2, 1\}$	0.0105
$\{7, 6, 5, 4, 3, 2\}$	0.0632
$\{7, 6, 5, 4, 3, 2, 1\}$	0.1956