Bounding analysis of lung cancer risk using imprecise probabilities

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Authors’ research interests

Granger M. Morgan, Lord Chair Professor and Department Head: Public policy; risk analysis; expert elicitation.

Elizabeth Casman, Research Engineer: Epidemiology, malaria, security.

Minh Ha-Duong, Chargé de Recherche, visiting: Uncertainty, environmental economics, global change, integrated assessment models.
Lung cancer
**Risk factors** $\Omega = \{C, R, A, X\}$

<table>
<thead>
<tr>
<th>$C$</th>
<th>$R$</th>
<th>$A$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smoking</td>
<td>Radon</td>
<td>Asbestos, fibers</td>
<td>All other</td>
</tr>
</tbody>
</table>

![Image of smoking cigarette](image1)

![Image of radon](image2)

![Image of asbestos fibers](image3)

![Image of laboratory equipment](image4)
Attribution is ambiguous

Multiple exposures...

Synergistic effects

\[
\begin{align*}
\bar{n}(C) &= n(C') \\
\overline{n}(C) &= n(C') + n(XC) + n(CR) + n(CA) = q(C)
\end{align*}
\]
Epidemiologist’s attributable fraction $af$

Fraction of deaths that could be avoided by eliminating exposure to a risk factor.

\[
\frac{n(C')}{N}(1 - r_0) \leq af(C') \leq \frac{\overline{n}(C')}{N}(1 - r_0)
\]

Studies in populations presumed to have low carcinogen exposure suggests a background lung cancer death rate $r_0$ around 3 for 100 000.
The rate of spontaneous lung cancer

Denoting $p_C = 0.45$, $p_R = 0.50$ and $p_A = 0.05$ the exposure probabilities to $C$, $R$, $A$ respectively, and assuming independance:

$$r_0 = \frac{n(\emptyset)}{(1 - p_C)(1 - p_R)(1 - p_A)T}$$

Observed number of cases is $N = 152.000$ out of $T = 270.000.000$ people.

$$n(\emptyset) = 0.013N$$
Method to infer $\overline{n}(X)$

1. Elicit known bounds on $C, R, A$ from experts
2. Find the most unspecific $n$ consistent with them

\begin{equation}
U(n) = \sum_{E} |E| n(E) \text{ for all subsets } E \text{ of } \Omega
\end{equation}

- Smets’ Transferable Beliefs Model
- *Mathematica* package
- Linear Programming
### Results

<table>
<thead>
<tr>
<th>Attributable fraction</th>
<th>C</th>
<th>R</th>
<th>A</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{a.f} )</td>
<td>95</td>
<td>21</td>
<td>5</td>
<td>3.2</td>
</tr>
<tr>
<td>( a.f )</td>
<td>70</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

| Exposure probability  | .45| .5 | .05| .05|

<table>
<thead>
<tr>
<th>Relative risk*</th>
<th>C</th>
<th>R</th>
<th>A</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{r.r} )</td>
<td>43</td>
<td>1.5</td>
<td>2.</td>
<td>1.66</td>
</tr>
<tr>
<td>( r.r )</td>
<td>6</td>
<td>1.</td>
<td>1.2</td>
<td>1.</td>
</tr>
</tbody>
</table>

*The relative risk of lung cancer deaths is \( r.r = \frac{\text{frequency in the exposed population}}{\text{frequency in non-exposed population}} \)
Conclusion

If one is confident in the bounds assigned to the well understood risk factors, no more than 5,000 people die from lung cancer induced by (relatively) minor pollutants, while the lower bound on smoking is 100,000.

Future research:

- Conduct the experts elicitation
- 3+ Pollutants interaction
- Measuring inter-experts consistency