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A hierarchical fusion of expert opinion in the Transferable Belief Model (TBM)

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The frame of reference: climate sensitivity

Climate sensitivity $\Delta T_{2\times}$ is:

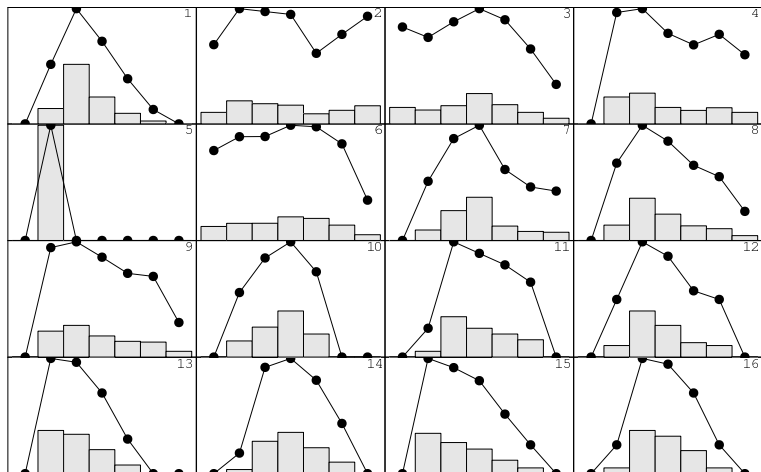
The long term global warming if $[\text{CO}_2]$ in the atmosphere doubles
Uncertain: 1.5°C to 4.5°C .

Morgan and Keith (1995) obtained probability density functions by interviewing 16 leading U.S. climate scientists.

Experts' uncertainty range subdivided in 7 intervals to simplify:

$$\begin{aligned}\Omega &= \{\omega_1, \dots, \omega_7\} \\ &= \{[-6, 0], [0, 1.5], [1.5, 2.5], [2.5, 3.5], [3.5, 4.5], [4.5, 6], [6, 12]\}\end{aligned}$$

Variety of views: everything possible $\{2,3\dots\}$, no cooling $\{4\dots\}$, reasonable middle $\{1\dots\}$, no problem $\{5\}$



Fusion issues using experts as information sources

- ▶ Dependence → Avoid unjustified accuracy
- ▶ Complete contradiction → Need paraconsistency
- ▶ Scientific validity \neq popularity → No majority rule
- ▶ Calibrating experts is not practical → don't !

Categorical beliefs: the indicator function $\mathbf{1}_E$

Belief that the state of the world is in the subset $E = \{\omega_2, \omega_3, \omega_4\}$

of the frame of reference $\Omega = \{\omega_1, \dots, \omega_7\}$ is represented by

$$m = \mathbf{1}_E$$

the indicator function of E :

$$\begin{cases} m(\{\omega_2, \omega_3, \omega_4\}) = m(E) = 1 \\ m(A) = 0 \end{cases} \quad \text{for any other } A \subset \Omega, A \neq E \quad (1)$$

Representing belief with a random subset of Ω

We allocate the unit “mass of belief” among subsets of Ω .

$m : 2^\Omega \rightarrow [0, 1]$ is a Basic Belief Assignment iff:

$$\sum_{A \subseteq \Omega} m(A) = 1 \quad (2)$$

Corner cases included: ignorance and contradiction

Total ignorance, no information Void beliefs represented by $\mathbf{1}_\Omega$.

Total confusion Contradictory beliefs represented by $\mathbf{1}_\emptyset$.

Discounting and simple beliefs

Discounting is adding a degree of doubt r to a belief m by mixing it with the void beliefs:

$$\text{disc}(m, r) = (1 - r) m + r \mathbf{1}_\Omega \quad (3)$$

Denote A^s the simple belief that
“The state of the world is in A , with a degree of confidence s ”:

$$A^s = \text{disc}(\mathbf{1}_A, e^{-s}) \quad (4)$$

That is:

$$\begin{cases} A^s(A) = 1 - e^{-s} \\ A^s(\Omega) = e^{-s} \\ A^s(X) = 0 \end{cases} \quad \text{if } X \neq A \text{ and } X \neq \Omega$$

Conjunction \odot and disjunction \oslash of beliefs

When two reliable information sources say one A and the other B , believe in the intersection of opinions (TBM allows $\mathbf{1}_\emptyset$):

$$\mathbf{1}_A \odot \mathbf{1}_B = \mathbf{1}_{A \cap B}$$

Generally:

$$(\mu_1 \odot \mu_2)(A) = \sum_{B \cap C = A} \mu_1(B) \mu_2(C) \quad (5)$$

When at least one source is reliable, consider the union of opinions.

$$(\mu_1 \oslash \mu_2)(A) = \sum_{B \cup C = A} \mu_1(B) \mu_2(C) \quad (6)$$

Canonical decomposition in simple beliefs

For any m such that $m(\Omega) > 0$, there are weights $(s(A))_{A \subsetneq \Omega}$ such that:

$$m = \bigodot_{A \subsetneq \Omega} A^{s(A)} \quad (7)$$

Weights of the \odot conjunction are the sum of weights:

$$m_1 \odot m_2 = \bigodot_{A \subsetneq \Omega} A^{s_1(A)+s_2(A)} \quad (8)$$

\odot Conjunction increases confidence: $A^s \odot A^s = A^{2s}$.

Good for independent information sources,
but for experts we want to avoid unjustified accuracy

T. Denœux's cautious combination operator

Whenever...

Expert 1 has confidence $s_1(A)$ that state of the world is in A

Expert 2 has confidence $s_2(A)$

...follow the most confident:

$$m_1 \hat{\otimes} m_2 = \bigcap_{A \not\subseteq \Omega} A^{\max(s_1(A), s_2(A))} \quad (9)$$

Distributivity: $(m_1 \odot m_3) \hat{\otimes} (m_2 \odot m_3) = (m_1 \hat{\otimes} m_2) \odot m_3$

Interpretation:

Expert 1 has beliefs $m_1 \odot m_3$

Expert 2 has beliefs $m_2 \odot m_3$

$\hat{\otimes}$ cautious combination of experts counts evidence m_1 only once.

Historical operators: Averaging and Dempster's rule

Averaging is $\frac{m_1(X)+m_2(X)}{2}$

Renormalizing m means replacing it with m^* such that $m^*(\emptyset) = 0$ and

$$m^*(X) = \frac{m(X)}{1 - m(\emptyset)}$$

Dempster's rule is renormalized conjunction:

$$m_1 \oplus m_2 = (m_1 \odot m_2)^* \quad (10)$$

There is no satisfying fusion operator

	Average	\oplus, \cap	\wedge	\cup
Majority rule	☹	✓	✓	✓
Contradiction	✓	☹	☹	✓
Unjust. accuracy	✓	☹	✓	☹

Discounting decreases contradiction issues,
but calibrating experts is not practical.

A hierarchical approach

1. Partition experts in schools of thought (adaptative or sociological methods)
2. Within groups, $\hat{\wedge}$ cautious combination
3. Across theories, $\hat{\cup}$ disjonction

Using the climate experts dataset:

m_A	$=$	$m_2 \hat{\wedge} m_3 \hat{\wedge} m_6$	Everything possible
m_B	$=$	$m_4 \hat{\wedge} m_7 \hat{\wedge} m_8 \hat{\wedge} m_9$	No cooling
m_C	$=$	$m_1 \hat{\wedge} m_{10} \hat{\wedge} \dots \hat{\wedge} m_{16}$	Reasonable middle
m_D	$=$	m_5	No problem
m	$=$	$m_A \hat{\cup} m_B \hat{\cup} m_C \hat{\cup} m_D$	

Probability and plausibility used to present results

Any m defines a probability p^m by:

$$p^m(\omega_i) = \sum_{X \ni \omega_i} \frac{m^*(X)}{|X|} \quad (11)$$

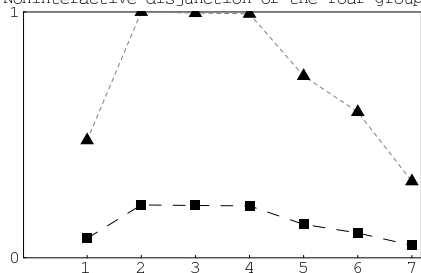
Any m defines a plausibility function pl ,
which is given on singletons by:

$$pl(\{\omega_i\}) = \sum_{X \ni \omega_i} m(X) \quad (12)$$

Levels of probability are generally smaller than levels of plausibility.

Results: fusion of 16 experts on $\Delta T_{2 \times}$, MK 1995 survey

Noninteractive disjunction of the four groups.



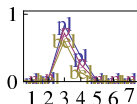
Simple distributions associated with the result BBA:

- ▲----- pl on singletons
- - ■ - - Pignistic probability

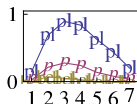
i	1	2	3	4	5	6	7
ω_i	-6,0	0,1.5	1.5,2.5	2.5,3.5	3.5,4.5	4.5,6.0	6.0,12
pl	0.48	1.	1.	0.99	0.74	0.59	0.31
p^m	0.08	0.21	0.21	0.21	0.14	0.10	0.05

Symmetric fusions operators vs. Hierarchical approaches

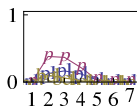
disc. Dempster



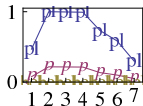
Averaging



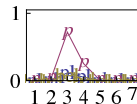
disc. Cautious conj.



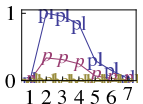
Hierarchical



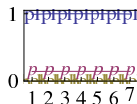
disc. niConj.



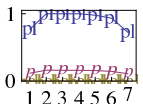
Hierarchical 3-way



niDisjunction



Average within



The likelihood of $\Delta T_{2x} < 1.5^\circ\text{C}$ has decreased since 1995

IPCC 2001: Climate sensitivity is likely to be in the 1.5 to 4.5°C range (unchanged from 1979).

$\Delta T_{2x} \in \dots$	$[0^\circ\text{C}, 1.5^\circ\text{C}]$	$[1.5^\circ\text{C}, 4.5^\circ\text{C}]$	$[4.5^\circ\text{C}, 10^\circ\text{C}]$
Published PDFs	[0, 0.07]	[0.31, 0.98]	[0.02, 0.62]
Kriegler (2005)	[0, 0.00]	[0.53, 0.99]	[0.01, 0.47]

IPCC 2007: [2, 4.5°C] is likely, below 1.5°C is very unlikely.

Note:

Likely means $0.66 \leq p \leq 0.90$,

very unlikely means $p \leq 0.1$.

Conclusions

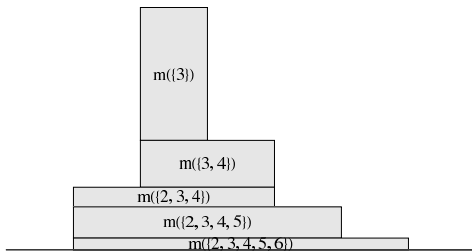
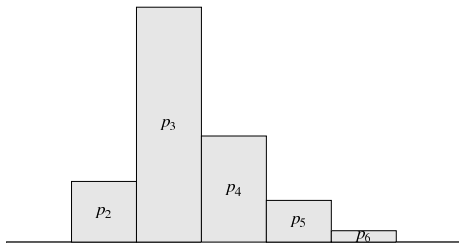
A hierarchical approach to fusion expert opinions:

- ▶ Imprecise
- ▶ Deals with dependencies and contradiction
- ▶ Avoid majority rule and calibration
- ▶ Requires a sociological study of experts groups

About climate sensitivity:

- ▶ Above 4.5°C was already plausible in 1995
- ▶ Below 1.5°C is less plausible today

Expert 1: bayesian m (top), consonnant m (bottom)

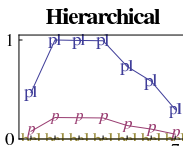
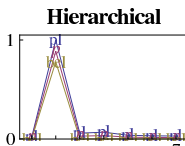
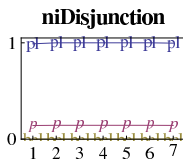
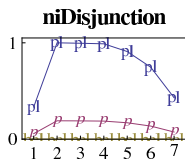
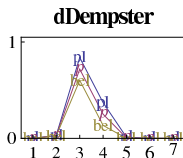
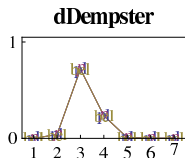
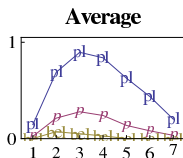
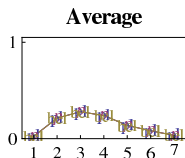


Hierarchical better than symmetric fusion for expert aggregation

	Average	\oplus, \ominus	\wedge	\cup
Majority rule	\ominus	✓	✓	✓
Contradiction	✓	\ominus	\ominus	✓
Unjust. accuracy	✓	\ominus	✓	\ominus

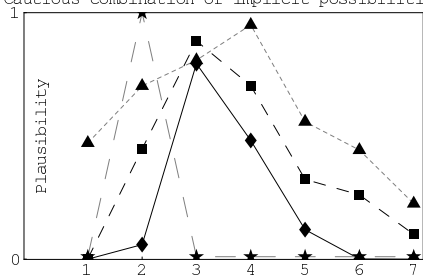
Fusion method	$m(\Omega)$	$\leq 1.5^\circ\text{C}$ <i>bel-pl</i>	In range <i>bel-pl</i>	$\geq 4.5^\circ\text{C}$ <i>bel-pl</i>
Hierarchical	0.18	0.-1.	0.-1.	0.-0.61
Average	0.08	0.07-0.69	0.27-0.93	0.-0.45
disc. Dempster	0.	0.02-0.03	0.97-0.98	0.-0.
Disjunction	0.99	0.-1.	0.-1.	0.-1.

Sensitivity analysis. Bayesian left, consonnant right.



Cautious combination within groups

Cautious combination of implicit possibilities



Experts groups:

- ▲--- 2,3,6
- -■- - - 4,7,8,9
- ◆— 1,10-16
- -★- - 5

Result of the hierarchical fusion: the belief function

subset A	$m^*(A)$
$\{2\}$	0.0001
$\{3, 2\}$	0.0074
$\{4, 2\}$	0.0033
$\{4, 3, 2\}$	0.1587
$\{4, 3, 2, 1\}$	0.0064
$\{5, 4, 2\}$	0.0011
$\{5, 4, 3, 2\}$	0.1321
$\{5, 4, 3, 2, 1\}$	0.0709
$\{6, 4, 3, 2\}$	0.0267
$\{6, 4, 3, 2, 1\}$	0.0129
$\{6, 5, 4, 3, 2\}$	0.0888

subset A (cont.)	$m^*(A)$
$\{6, 5, 4, 3, 2, 1\}$	0.1811
$\{7, 4, 3, 2\}$	0.0211
$\{7, 5, 4, 3, 2\}$	0.0063
$\{7, 6, 4, 3, 2\}$	0.0135
$\{7, 6, 4, 3, 2, 1\}$	0.0105
$\{7, 6, 5, 4, 3, 2\}$	0.0632
$\{7, 6, 5, 4, 3, 2, 1\}$	0.1956